Visual Guidance for Fixed-Wing Unmanned Aerial Vehicles using Feature Tracking

Application to Power Line Inspection

by

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- Skid-to-Turn Manoeuvres
- Forward-Slip Manoeuvres
- Feature Tracking
- Vision Based Control
- Position Based Visual Servoing
- Image Based Visual Servoing

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The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

Steven John Mills
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Abstract

As the use of Unmanned Aerial Vehicles (UAVs) grows within the civilian sector, one application that is likely to attract the attention of industry is the inspection of infrastructure, in particular, those situated in rural and remote regions. Automating the process of data collection would appear to be a task well suited to the UAV and one that can draw upon years of research in areas of machine vision, guidance and control, and automated data processing. Fixed-wing UAVs can be expected to play a crucial role in this, particularly for tasks covering large areas, due to the platforms inherent efficiency and generous payload capabilities that directly contribute to long range.

Successful completion of these tasks introduces the challenge of performing guidance and control in a manner that establishes favourable conditions for data collection. While various tracking solutions exist, a common approach is to guide the vehicle directly over the feature that inevitably sees data collection controlled indirectly as a by-product of aircraft position. In particular, these solutions overlook sensor line-of-sight that is directly affected by aircraft attitude that varies as a result of rotation induced by manoeuvres used to maintain track. In the context of downward facing sensors that are likely to be fitted to fixed-wing UAVs, the impact is most evident through Bank-to-Turn manoeuvres that form the predominant means of altering heading.

Current solutions addressing these issues are limited and generally seek to address the problem through path planning and following that assumes knowledge of infrastructure location. Obtaining this information at a level of accuracy that can take advantage of these techniques however is not always possible. In this work, solutions are presented in the form of vision based control, offering real-time control capable of actively tracking infrastructure. Guidance and control is developed on the principal of providing ideal conditions for data collection from
Abstract

Body-fixed sensors, removing the need for gimballed mounts and thus alleviating payload requirements that are crucial on small UAV systems. Utilising Image Based Visual Servo (IBVS) techniques, data collection is controlled directly as viewed from an inspection sensor; a technique that is then extended to provide coverage as the UAV transitions between segments of locally linear infrastructure.

In the first of two developments, Skid-to-Turn (STT) manoeuvres are utilised through an IBVS control design to view the feature at a Desired Line Angle, calculated as a function of Sensor Track Error, that allows recentring of the feature in one smooth motion. The second development augments the interaction matrix of a line feature with the aircraft equations of motion. This allows the design of an optimal state feedback controller that enables tracking to be performed through Forward-Slip (FS) manoeuvres. These manoeuvres are shown to improve tracking performance at reduced control effort compared to STT, while control through state feedback provides a direct means to suppress unwanted motion that could otherwise degrade data collection.

Another contribution is made to the direct management of data collection through an analysis of visual tracking in the presence of wind. To track a desired course in the presence of wind requires heading to be altered by a Wind Correction Angle. This presents an issue for visual control formed on a desired view of features that does not account for wind. The issue is investigated through the inclusion of a wind model in the interaction matrix, linking relative motion of image features with aircraft motion and wind. The effect of a steady wind disturbance is found to introduce a constant term in the interaction matrix and shown to be offset with desired line angle set to the Wind Correction Angle.

A final contribution extends these developments to negotiating transitions between locally linear segments of infrastructure. Transitions present discrete changes in the direction of infrastructure that require a UAV performing inspection to alter course whilst ensuring continued data collection. Both the STT IBVS and FS IBVS developments are extended to this task, the first using a smoothing feature to manage the transition, while the latter switches between features at a predetermined distance in the image frame. These provide separate solutions with variations in overshoot, time to recentre and maximum transition angle.

Each of these developments is tested extensively through simulation, in an environment developed to generate imagery as would be captured during inspection, while allowing realistic test conditions including turbulence and wind gusts.
## Acknowledgements


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List of Abbreviations

AGL  Altitude above Ground Level
AOA  Angle of Attack
ARCAA  Australian Research Centre for Aerospace Automation
BTT  Bank-to-Turn
c.g.  Centre of Gravity
CRCSI  Cooperative Research Centre for Spatial Information
DOF  Degrees of Freedom
ECEF  Earth Centred Earth Fixed
FOV  Field of View
FS  Forward-Slip
GIS  Geographic Information System
GPS  Global Positioning System
IBVS  Image Based Visual Servoing
IMU  Inertial Measurement Unit
INS  Inertial Navigation System
LiDAR  Light Detection and Ranging
LoS  Line of Sight

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<tr>
<td>LLA</td>
<td>Latitude, Longitude, Altitude</td>
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<td>LQR</td>
<td>Linear Quadratic Regulator</td>
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<tr>
<td>MP</td>
<td>Megapixel</td>
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<tr>
<td>NED</td>
<td>North, East, Down</td>
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<td>PBVS</td>
<td>Position Based Visual Servoing</td>
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<td>PID</td>
<td>Proportional-Integral-Derivative</td>
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<td>PN</td>
<td>Proportional Navigation</td>
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<tr>
<td>QUT</td>
<td>Queensland University of Technology</td>
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<tr>
<td>RoC</td>
<td>Rate of Climb</td>
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<td>STT</td>
<td>Skid-to-Turn</td>
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<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
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<td>UGV</td>
<td>Unmanned Ground Vehicle</td>
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<td>VTOL</td>
<td>Vertical Take-Off and Landing</td>
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The desire to automate and control processes, from the everyday of performing a reverse park, to controlling the orbit of the Apollo space missions, has provided the inspiration and motivation for some of the most exciting developments over the last century. While the use of robotics in assisting everyday tasks has to a greater extent been confined to those fixed or constrained to operation on the ground, the development of Unmanned Aerial Vehicles (UAVs) has seen steady growth in the military sector since the late 1950s that now sees it at a point to be adopted by industry for civilian applications [1]. Particularly as the cost of the systems decrease, and consumer off-the-shelf products become readily available, it can be expected that such systems will be quickly taken up by industry.

Often noted for their suitability in dull, dangerous and dirty tasks [2], many of the emerging developments in UAV technology for civilian applications concern their use in surveillance, inspection and observation roles [3–5]. The choice of platforms used to fulfil these roles can be broadly classified between fixed-wing, rotorcraft and airships, with each offering unique attributes that can be exploited for various aspects of the task. Specifically designed to hover and manoeuvre at low airspeeds, rotorcraft including helicopters, quadrotors and ducted fans
have the potential to fulfil detailed inspection and observation roles that may include viewing the feature from multiple angles and maintaining position while an operator assesses live video footage. For tasks that require longer endurance with the UAV operating within a confined area, an airship may offer a sound alternative as the platform is able remain airborne for extended periods of time whilst providing a stable base for inspection sensors. For those inspection tasks that require the UAV to cover large distances, then a fixed-wing platform may offer a preferable solution, flying at higher airspeeds and capable of carrying larger payloads, the fixed-wing platform can offer efficient coverage over long distances.

Flying above the feature offers many benefits, including a generally unobstructed view, while avoiding common obstacles faced by ground inspection including fences, rivers, highways and difficult access routes. With very little traffic at the altitudes at which the UAVs would operate (e.g. below 400 ft), the UAV is also likely to avoid interfering with manned aircraft, while developments in sense and avoid and force landing will ensure the safety of the general public [6, 7]. While many of the sensors equipped on manned aerial inspection vehicles are infeasible for small UAVs given their size, weight and cost, there still exists ever expanding array sensors that become available as technology advances and sensor size and weight reduces. This enables potential inspection UAVs to collect a range of data including images and video, in a range of spectrums including visual, infrared and multi-spectral, while active sensors including LiDAR, Millimeter-Wave Radar (MMR) and ultrasonic become increasingly more accessible.

Enabling onboard sensors to collect data during tracking is addressed through a guidance and control algorithm, for which the development of such algorithms has seen steady interest over the past decade. While applications of road following [8–12] and power line inspection [13–17] have been key motivators, techniques are equally applicable to the inspection and observation of pipelines [18], rivers [19], coastlines, borders [20] and forest fire boundaries [21]. A common approximation for such tracking tasks is to consider infrastructure as locally linear, modelling the feature as a series of piecewise linear segments, that can be considered from the perspective of guidance and control as straight path following. Although successful tracking is reported by many, performance is commonly assessed in terms of Cross Track Error that provides a measure of perpendicular distance between vehicle and feature centreline. In other words, the tracking task is posed from the perspective of flying directly over the feature, as opposed to maintaining
sensor line-of-sight (LoS) on the target.

Although this assumption may hold for rotorcraft and airships that can manoeuvre through force vectoring, it can overlook serious issues of data capture from fixed wing UAVs that perform body rotations in many of the manoeuvres used to alter heading [22]. For example, performing a Bank-to-Turn manoeuvre that is the common means of altering aircraft heading, directly rotates downward facing sensors away from horizontal, subsequently angling them away from the direction of the turn. In the case of a UAV manoeuvring towards a feature, the result is the sensor being angled away from the feature, which is likely to already be off-centre prior to the manoeuvre (hence the need to perform it), risking the feature leaving the sensor field-of-view altogether.

In addition to ensuring the feature remains visible during tracking, another important requirement during inspection is ensuring motion of the sensor is minimal to avoid corrupting or degrading captured data [23,24], an issue commonly overlooked in tracking. Rapid rotation of the sensor can lead to a swift panning motion of the sensor, that can easily degrade data to the point where features can no longer be detected and subsequently results in missing coverage over these sections of the inspection area. Minimising motion not only addresses this issue, but also serves to improve overall data quality throughout the inspection process. Ideally a tracking controller would seek a balance between centring the feature within the sensor FOV and minimal sensor motion, thus providing optimal data collection conditions.

One means of addressing both of these issues would be to incorporate the inspection sensor into a gimballed mount, allowing the sensor to be decoupled from aircraft rotation and controlled in such a way to cancel unwanted motion, or further extended to track the feature. Although this can provide an effective means of orientating the sensor towards the feature, physical restrictions of the gimbal limit the range of compensation available, while the resulting view of the feature can be less than ideal as it is likely to appear at an angle as opposed to a direct overhead view. Furthermore, as the number of inspection sensors increase, so does the complexity of mounting multiple sensors in a single gimbal. An example of this includes the inspection for vegetation encroachment surrounding power lines, where techniques utilise a combination of multi-spectral imagery to first detect trees within the power line corridor, followed by an analysis of LiDAR or Stereo Imagery to estimate the height relative to power lines [25].
The development of a real-time tracking solution that enables simultaneous tracking and data collection would greatly benefit UAVs in a wide range of inspection and surveillance tasks. Adopting alternative manoeuvres that avoid unwanted motion not only assists the sensor used for control but also any other sensors that are equally aligned onboard the UAV. Payload requirements are reduced given the ability to mount sensors body-fixed, while also offering to reduce the overall compensation required for those with gimballed mounts. Providing a solution through real-time tracking also enables the system to correct for any errors during tracking that would otherwise arise given position estimate errors of the aircraft or errors in the known location of infrastructure.

Current advances in visual control, in particular visual servoing, offer opportunities for novel solutions that address many of the issues surrounding simultaneous tracking and data collection, including direct control of feature view, suppression of unwanted motion and real-time tracking. Although such a vision based approach will inevitably increase computational requirements onboard the UAV, the benefits of introducing such a system would prove invaluable to a wide range of applications that would likely drive the development of specific hardware to achieve the task.

1.1 Research Problem

The overall objective of this research focuses on the development of guidance and control algorithms enabling fixed-wing UAVs to track locally linear infrastructure while providing optimal conditions for data collection from downward facing, body-fixed sensors. In particular, emphasis is placed on developing techniques that allow the inspection task to be controlled directly in terms of obtaining a desired view of infrastructure, which given a body-fixed sensor, requires control of aircraft attitude to control sensor field-of-view. The envisaged result is a set of guidance algorithms that provide simultaneous tracking and data collection, utilising manoeuvres that assist data collection to ensure infrastructure is well position within the field-of-view of inspection sensors.

In order to fulfil this objective, the research posed three questions that would investigate and address underlying concepts and issues that would lead to satisfying the overall objective. From the perspective of improving data collection,
a key issue is avoiding motion that sees features move rapidly within the sensor field-of-view, that in the worst case can see features leave the field-of-view altogether. This leads directly to the first research question:

**Question 1:** *Can a fixed-wing UAV be controlled in such a way to provide tracking of locally linear infrastructure whilst enabling data collection from a downward facing sensor?*

Fundamentally the question concerns the ability to adopt alternate manoeuvres that avoid motion that would otherwise degrade data collection. In particular, manoeuvres that allow a fixed-wing UAV to simultaneously alter course, and thus ensure continued tracking, while minimising, or even assisting the sensor line-of-sight for the purpose of data collection. Utilising these manoeuvres in conjunction with visual information obtained from a downward facing image sensor would then provide a means for real-time tracking, leading to the second research question:

**Question 2:** *Can visual information extracted from a downward facing imaging sensor be utilised to control a fixed-wing UAV in capturing a desired view of infrastructure?*

This assumes infrastructure has been detected and extracted from imagery, and is concerned with utilising the information to control the UAV such that the feature is subsequently viewed at a desired position and orientation that is ideal for inspection. Addressing this issue fundamentally alters the perspective of tracking control from one of flying directly over the feature, i.e. minimising Cross Track Error, to tracking the feature from within the sensor field-of-view.

Having enabled real-time tracking from the perspective of data collection, the final step in fulfilling the research objective is addressing transitions between segments, leading to the third and final question:

**Question 3:** *Can techniques be developed from visual cues alone that enable the UAV to successfully negotiate the transition between segments while ensuring continued data capture?*

Transitioning between segments represents a uncertain period in the tracking process where defining a desired view, and controlling the UAV to maintain it, is somewhat arbitrary as both the current and future segments are in the sensor
field-of-view. At this point, the concept of an inspection *corridor* is introduced that describes an area surrounding infrastructure that is to be included in data capture. The concept is introduced as a means of assessing performance during transitions, where maintaining the full width of the corridor in the sensor field-of-view during tracking provides clear indication that the infrastructure is well positioned during data capture, where otherwise specifying the feature to remain ‘centred’ no longer provides a clear measure of performance.

Handling the transition itself based on visual cues obtained during real-time tracking presents a number of challenges, from determining when and how to transition, to dealing with the limited time in which the controller would have to initiate the transition. Closely related to this is establishing the threshold at which the UAV would no longer able to adequately perform a transition between segments given the angle of intersection. At this point the UAV would then be commanded to perform an alternate manoeuvre, making a circuit away from the infrastructure such that on the second approach the heading is suitable to resume tracking, also known as a ‘clover-leaf’ or ‘go-around’ manoeuvre [26,27].

### 1.2 Contributions of Research

The original contributions this thesis makes to knowledge concern the development of enabling technologies to improve data collection during tracking for fixed-wing UAVs. Specifically, the thesis makes a total of four main contributions which are described below.

A first contribution is made through a novel Skid-to-Turn (STT) Image Based Visual Servo (IBVS) controller, utilising Wings-Level STT manoeuvres through an IBVS control design to enable tracking of linear infrastructure while simultaneously providing improved conditions for data collection. The design introduces the concept of tracking from the perspective of the inspection sensor, allowing control to be posed directly in terms of data collection. Utilising concepts of Image Based Visual Servoing, the controller commands Skid-to-Turn manoeuvres to obtain a *Desired Line Angle* calculated based on current *Sensor Track Error*. Performance was assessed against common techniques and shown to provide improved conditions for data collection, obtaining a desired view while reducing feature motion by an average of 80%.
1.2 Contributions of Research

A second contribution is made in the form of a Forward-Slip (FS) Image Based Visual Servo (IBVS) controller, that fully utilises the dynamics of the platform to offer improved performance at reduced control effort. The design linearises the interaction matrix of the extracted feature and then augments this with the lateral dynamic equations of motion from which a full state feedback controller is developed using an optimal LQG control design that accommodates reduced measurement rates from feature extraction. The design not only allows direct control of feature position and orientation, but also enables a direct approach to the suppression of unwanted motion. The resulting controller utilises Forward-Slip manoeuvres and is shown to increase performance by 67% using similar levels of control effort to Skid-to-Turn, while reducing maximum control surface deflections by over half.

A third contribution is made in the development of a control solution for improving data collection where an estimate of mean wind conditions is available. The development introduces a model of mean wind to the interaction matrix of the extracted feature and through linearisation is shown to satisfy steady state tracking with the introduction of the Wind Correction Angle. The design is shown to improve performance over original compensation techniques in winds up to 37 km/h (20 kn), with the effects of turbulence on sensor line-of-sight shown to be minimal, while providing adequate suppression of isolated gust disturbances up to 50% of mean wind conditions and sustained gusts up to 20%.

A fourth contribution is made through a solution to negotiate transitions between segments of locally linear infrastructure through visual cues alone. The solution connects the point of entry of new segments entering the image frame with the exit point of the current segment, providing a temporary line for tracking that gradually shifts emphasis between the two as they move through the frame. The solution is shown to provide smooth transitions that enable data collection, where limits are found to be directly linked to sensor footprint that effects the distance at which the transition is detected. Performance is shown to improve with increased sensor footprint that can be achieved by increasing operating altitude or utilising sensors with wider FOV, where an auxiliary inspection sensor with narrow FOV can operate simultaneously to improve coverage.

Each of these developments is demonstrated through simulation, within an environment specifically developed for the task. While demonstration through flight testing would have been ideal, the resources to perform such a series of
experiments were unfortunately not available. Aside from the basic need for access to a fully operational UAV system with autopilot and ground station, the developments call upon onboard image processing and feature extraction capable of detecting infrastructure in real-time. At the time of this research, such a system was not available, and the work load to design, implement, test and integrate such a system onto a UAV was deemed beyond the scope of this research. In place, time was spent ensuring accuracy and fidelity of the simulation environment was preserved in light of the manoeuvres performed.

1.2.1 Publications

The following is a list of papers published over the course of the research program:


  *Best Student Paper Award, UAV’10 Conference*


1.2 Contributions of Research

1.2.2 Significance of Research

Aside from the contributions this research makes to the field of research, it is also expected that the outcomes of this thesis could offer real world benefits, particularly given the practical nature of problems its sets to address. More so given that the research aligns directly with the objectives of a cooperative research project that is part of the Cooperative Research Centre for Spatial Information (CRCSI) with industry partners Ergon Energy, Australian Research Centre for Aerospace Automation (ARCAA) and the Queensland University of Technology (QUT). As the Australian state of Queensland’s largest supplier of electricity, Ergon Energy has in excess of 150,000 km of power lines, supported by over 1 million power poles, covering an area of 1.5 million square kilometers, resulting in one of the widest spread networks in the world [25].

Under government regulation, electricity suppliers are required to fulfil strict maintenance schedules that require the inspection of infrastructure at regular intervals. As such, Ergon Energy foresee significant benefit in technology that can automate any stage in the process, from data collection to automated processing. In terms of monitoring vegetation alone, Ergon Energy currently spends $80 million a year inspecting and managing vegetation that encroaches on power line assets, where close to $10 million of that is spent on preliminary inspection to identify areas requiring closer assessment, just one of the many roles that would be well suited to aerial inspection from UAVs. This research has been directly linked with Project 6.7 of the CRCSI that sought enabling technology for this problem and thus the outcomes of this research would be expected to provide a direct contribution.

In a broader sense, this research addresses issues faced by many applications requiring data collection including the inspection of oil and gas pipelines, railways and borders, while also assisting the monitoring of rivers, coastlines and forest fires. Even assessing the feasibility, or planning routes, could benefit from the outcomes of this research in terms of providing an in-depth investigation of turning requirements that enable data collection.
1.3 Scope and Key Assumptions

The scope of this research considers:

- Fixed-wing platforms with rudder control.
- Platforms of a hobby, radio controlled size. Anticipated weight would be approximately $5 \sim 30$ kg, with wingspans ranging between $2 \sim 6$ m.
- Operation within an airspace free of obstacles and obstructions.

Key assumptions made throughout this research include:

- The presence of a feature extraction algorithm that can process captured imagery.
- A pre-existing autopilot that enables the UAV to be guided to the inspection zone, with provision to maintain altitude, airspeed and wings-level on command.
- Body-fixed mounting of inspection sensors, each in a downward facing orientation with equal alignment with the body axes of the UAV.
- Known location of infrastructure to an accuracy that allows the UAV to navigate within range for initial visual detection.

A general note should be made as to the role of this research within an envisaged system architecture. The intention is to provide guidance and control for tracking features when the UAV is already in position to begin data collection, i.e. infrastructure is within the field-of-view of an onboard imaging sensor. This requires the UAV to have systems in place that provide navigation to this location; a location that itself will be determined by a route management system controlling the overall inspection task.

Likewise, in the event that vision based tracking fails, it would be assumed that the higher level navigation system would retake control; disabling the vision based control system until the UAV had recovered and could resume data collection. This could include, failing to detect features, missing sections of the inspection area due to wind, or failing to negotiate a transition. In each case it would be assumed that a higher level navigation system will be available to recover the UAV and re-establish the UAV in a location that allows vision based tracking, and thus data collection, to recommence.
1.4 Thesis Outline

This thesis is structured as follows:

Chapter 2 provides an in-depth review of literature concerning those areas of most relevance to this research. This includes UAV path planning and path following, particularly in the context of inspection and surveillance performed by fixed-wing UAVs. This is followed by an in-depth review of current developments in visual control of UAVs and general vision based control techniques that were identified as providing potential paths to addressing the research objectives.

Chapter 3 presents a novel Image Based Visual Servo control design that utilises fixed-wing aircraft manoeuvres that eliminate unwanted motion to reduce both the time taken to recentre the feature and overall motion of the feature as viewed by an inspection sensor. The chapter begins with a discussion of issues surrounding current techniques used in tracking and then details the development of the visual controller. Testing is performed in simulation under a range of conditions including wind and varying operating parameters, with response compared to that of a generic solution approximating the common approach presented in literature for visual tracking, utilising position based techniques executed through Bank-to-Turn manoeuvres.

Chapter 4 presents another novel Image Based Visual Servo control design, through a development that allows the controller to fully utilise the dynamics of the system to the advantage of improving data collection during tracking. The chapter begins with the development of the interaction matrix for the extracted feature that is then linearised and augmented with the linearised equations of motion of the aircraft. Control and observer development are then detailed, followed by a series of simulations testing the proposed controller over a range of operating parameters.

Chapter 5 presents a novel solution for vision based tracking given an estimate of mean wind conditions that offers considerable improvement in data collection conditions over existing techniques. The chapter begins with a model of mean wind and incorporates this into the development of the interaction matrix for the extracted feature that leads to introducing the Wind Correction Angle into the interaction matrix. This is then incorporated into the control design of Chapter 4. Response of the new design is tested under both known wind conditions and unknown disturbances in the form of turbulence and gusts.
Chapter 6 presents a novel solution to negotiating transitions in locally linear infrastructure while preserving data collection over these regions. The chapter begins with the development of a smoothing function that allows a continuous transition from the perspective of tracking. This is then tested with the developments of both Chapters 3 and 4 and includes a series of test cases to assess the limits of such transitions. The chapter concludes with an example of data collection performed over a simulated model over known infrastructure, including inaccuracies in known versus actual location that heavily impact data collection, and ultimately highlight the benefit and contributions of this research.

The thesis concludes in Chapter 7 with a summary of contributions and a discussion of future work that could follow on from this research. Appendix A details a number of underlying mathematical derivations used in the developments of this research, while Appendix B provides a detailed description of the simulation environment in which outcomes of the research were assessed.
Automating the inspection of infrastructure using Unmanned Aerial Vehicles (UAV) can be considered three supporting, although typically independent, tasks that begin with route planning, executed through path following and analysed through data processing. In the field of mathematics, both exact and heuristic solutions to the routing problem have been sought since the 18th century, dating back to the “Königsberg Bridge Problem” solved by Euler in 1736 [28]. The problem posed whether the seven bridges crossing the river Pregel in the town of Königsberg (as illustrated by Euler’s sketch in Figure 2.1a) could be crossed, each only once, in a closed walk [29].

Euler was able to prove that no such path existed, having posed the problem as an undirected graph, as illustrated in Figure 2.1b, from which he was able to show that for a solution to exist, the graph must be connected, with each of the vertices of even degree, a property now known as Eulerian. An extension of this problem then becomes one of finding the shortest path that traverses all edges of a non-Eulerian graph, a problem first posed by the Chinese mathematician Guan in 1962 for route selection of a postman [30], for which the problem is now more commonly known as the Chinese Postman Problem (CPP) [31].
In much the same way that the famous Travelling Salesman Problem (TSP) addresses the shortest closed path to visit each node or vertex of a graph [32], the CPP seeks to find the shortest closed path that traverses each edge or arc. For a non-Eulerian graph the problem becomes one of graph augmentation, replicating edges to create a minimum distance path, or least cost, in the case of the Arc Routing Problem (ARP) that generalises the CPP, considering constraints in addition to distance [33].

While the application of such techniques to UAV flight path planning for inspection introduces a number of key challenges and constraints (e.g. minimum turn radius [34]), the problem can draw upon extensive research and variants of the classical problem that include:

- The “Windy Postman Problem”, that addresses the penalty/gain of traversing an edge against/with the wind respectively. [35]
- The “Rural Postman Problem”, where only a subset of edges must be traversed, with remaining edges providing links and possible shortcuts. [36]
- And “Capacitated Arc Routing”, where route selection considers the minimum cost traversal of all edges utilising $m$ capacity constrained vehicles, each required to travel a closed path. [37,38]

Once a desired path has been selected for the UAV, attention may then turn to data collection. A wide selection of sensors are available for inspection including Imaging Sensors (High Resolution, Infrared (IR), Ultraviolet (UV), Multi-spectral, Stereo Vision) [27,39,40], Corona Detection Devices [41,42] and Active Sensors (Light Detection and Ranging (LiDAR) [25], Millimeter Radar
(MMR) [43]), a selection that is likely to grow as technology improves and sensor size decrease. While the selection of sensors will vary between applications, a common requirement is ensuring the Field of View (FOV), or sensor footprint, of each sensor remains on the infrastructure for the full length of the inspection route. This places an additional requirement on the whole process that must be considered at both the path planning and path following stages to ensure simultaneous tracking and FOV control is achieved.

2.1 Path Planning

One of the most simple and common navigation techniques implemented on UAVs is that of Waypoint Navigation [44]. During Waypoint Navigation, a navigation controller provides the autopilot with a target location, or waypoint, to fly towards that is used to calculate a desired heading given the global position of the UAV. A waypoint is then considered captured once the UAV reaches the target location (generally within a predefined distance to account for any tracking errors), triggering the next waypoint in the sequence that continues until the UAV reaches its final destination. Utilising Waypoint Navigation for the purpose of path following is then achieved through selection of a sequence of waypoints placed along the original path, approximating it as piecewise linear with waypoints defining major turning points [45].

Although the approximation of inspection routes as piecewise linear is generally applicable to manned made infrastructure (e.g. power lines, pipelines, borders), waypoint navigation is still liable to yield poor results for fixed wing UAVs as it places no objective on tracking the segment between waypoints. As a result, the final path taken by a UAV capturing a sequence of waypoints can differ greatly from the piecewise linear path, as illustrated in Figure 2.2, particularly in the presence of disturbances, e.g. wind [46]. Introducing lateral track control to follow the adjoining segments of a waypoint path can reduce these issues, and is an approach taken by Niculescu, calculating desired ground velocity vectors for an existing autopilot to intercept the linear segment at a distance proportional to the current waypoint [47], and by Kang and Hedrick through Nonlinear Model Predictive Control (NMPC) that allows optimal trajectories to be generated for smooth and timely convergence on the path [48].
Transportation departments in Ohio

Figure 2.2: Tracking issues associated with the use of waypoints for following roads, pipelines and power lines as illustrated by Rathinam, Kim and Sengupta [46]

While lateral track control provides a measure of path following between waypoints, the heading at which the aircraft arrives and leaves each waypoint inevitably leads to overshoot in the transition given the limited turning radius of the aircraft [26] that can lead to inspection sensors missing sections of the path [49,50]. Implementing a look ahead, proximity or switching distance to initiate turns earlier can address these issues, as is presented by Kang and Hedrick in [48] where line segments are switched at a distance that accommodates the minimum turn radius of the aircraft. The minimum turn radius in this case however is calculated under ideal conditions that can be effected by variations in airspeed and outside disturbances including wind. These issues are addressed by Osborne and Rysdyk in [49], where a proximity distance for initiating transitions earlier is calculated to not only account for minimum turn radius but also factor in change in course angle, airspeed and windspeed, providing improved convergence and reduced overshoot to that of minimum turn radius alone.

An alternate approach is to alter the path itself to accommodate for vehicle kinematics thus producing a flyable path that ensures the vehicle can pass through waypoints, a process better known as trajectory smoothing, that has been a topic of interest in autonomous vehicles for a number of decades dating back to two degree of freedom Wheeled Mobile Robots (WMR) [51–53]. Many of the techniques developed for WMRs have subsequently been applied to trajectory smoothing of UAV paths and can generally be divided between spline-based methods [54–58] and Dubins principles [59–63]. In 1957 Dubins showed that the shortest path between any two points constrained by intial and final orientations is given by a
path constructed of straightline segments and circles of maximum curvature [64], thus assuming an aircraft operates at a fixed airspeed and constant altitude the minimum turn radius will remain constant and can be used to construct a Dubins path that is dynamically feasible for the aircraft. While Dubins paths are simple to construct and implement, the curvature of the path is only $C^1$ continuous, in other words, while the path has no sudden changes in direction ($C^1$ continuous), it does have discontinuous changes in curvature ($C^2$ discontinuous) that leads to instantaneous changes in commanded acceleration at the segment boundary of arcs and straight path segments [65,66].

Spline curves offer a solution to this problem offering $C^2$ continuity and have been proposed in a number of forms to address the path planning problem for Fixed Wing UAVs. Shanmugavel et al. propose a solution that replaces the circular arcs of Dubins paths with Clothoid curves, utilising their linear curvature profiles to remove the discontinuity between circle-line-circle segments of the Dubins path to produce a smooth transition and $C^2$ continuity [67]. Controlling the shape and end point conditions of Clothoid segments is however considered non-trivial by many authors [68, 69], as the curves have no closed form expression, requiring numerical integration techniques for evaluation. Bézier curves offer a parametric solution that has made them a popular alternative for trajectory smoothing, with applications to UAV path planning including the work of Yang and Sukkarieh and the use of cubic Bézier spiral curves to generate paths that closely adhere to the original piecewise linear path to avoid obstacles in a cluttered environments [57]. Pythagorean-Hodograph curves also offer a parametric solution with the useful property of expressing arc-length, curvature and offset as a set of rational functions that allows control of path length that has been applied to path planning for the simultaneous arrival of multiple UAVs [70,71].

Although the paths generated by these algorithms provide feasible trajectories for the UAV to fly, the paths are generally designed with the goal of flying from ‘Point A’ to ‘Point B’, where the waypoint path is developed to avoid obstacles and threats [72]. This places less emphasis on following the original piecewise linear path, generally relaxing the constraint on passing ‘through’ waypoints, in favour of smoothing corners to produce the shortest path, or relaxing the need to fly linear segments (Figure 2.3). In the context of inspection, following the waypoint path is the most significant aspect of the task and represents the goal of the mission, as opposed to reaching ‘Point B’ in the shortest time [16].
Creating feasible paths that adhere to an original piecewise linear waypoint sequence is a topic addressed in the masters dissertation of Anderson [73], for which resulting techniques were later published in [60] and implemented by Kingston [74] and Beard et al. [75]. The original solution developed by Anderson in [73] seeks to minimise deviation from an original waypoint path through the adoption of dynamically feasible, time extremal, $\kappa$-trajectories. The $\kappa$-trajectories proposed utilise three sets of circular arcs that initiate three opposing turns, each of which meet the minimum turn requirements of the UAV, and allow control over the path taken in capturing each waypoint. Three examples of waypoint capture are proposed, as illustrated in Figure 2.4a, including preserving the distance travelled to that of the original path, ensuring waypoint capture is preserved with the UAV passing through the waypoint at the apex of the turn, and minimising the time to traverse the path, simplifying the solution to that of a Dubins path.
2.1 Path Planning

![Diagram](image)

(a) Anderson $\kappa$-trajectories path planning examples [60]

(b) ‘Go-Around’ or ‘Clover-leaf’ manoeuvre [27]

Figure 2.4: Waypoint capture for vehicles with limited turn rate.

Alternatively, at the cost of extra flight distance and assuming no obstacles lie in the surrounding area, full coverage of the flight path and waypoints can be guaranteed through a ‘go-around’ manoeuvre [27], or ‘clover-leaf’ flight path [26], that effectively sees the UAV pass each waypoint twice, illustrated in Figure 2.4b. Rather than initiating a turn early, the UAV is left to complete the segment before performing an opposing turn, in which time, the UAV adjusts heading to align with the next segment before passing the waypoint for a second time. This allows full coverage of both segments, although given the additional distance added to the flight path, would be reserved for when aircraft turn rate is deemed insufficient to make the transition between segments through a normal turn. The technique not only offers to provides complete coverage but provides two opportunities, at two separate angles, to inspect any object that coincides with waypoint (e.g. power poles, intersections).

2.1.1 Path Following

Once a path has been planned to navigate the inspection route, guidance and control of the UAV to execute the path is achieved through a path following, or in
the case of time critical missions, trajectory tracking algorithm. In the domain of autonomous vehicles, the term trajectory tracking generally refers to the notion of controlling vehicular position as a function of space and time [76]. In other words, the trajectory not only defines the path as a function of vehicle location, but desired timing for which the vehicle should traverse the path. [77]. For dynamically constrained vehicles, including the fixed-wing UAV, such dynamics necessitate the trajectory define not only position but orientation to ensure a feasible path [78], for which control becomes one of tracking a “state-space” time-parameterised trajectory [79].

For tasks where the vehicle is only required to be on-path, generating time-parameterised paths is not only added complication, but can lead to poor performance due to outside disturbances, including wind that can introduce accumulating errors as aircraft groundspeed differs from indicated airspeed [48, 80]. In these instances, the tracking problem can be reposed as one of path following, a variation of guidance and control that negates the timing constraints imposed by trajectory tracking [81]. As an integral process in automated flight, the development of path following algorithms for UAVs has seen considerable research, and while control techniques differ, algorithms can generally be divided between four strategies; Cross Track Control [82], ‘Good Helmsman’ Behaviour [49, 83], Vector Fields [50, 81, 84] and Virtual Targets [85–91].

The term Cross Track Control is used here to classify controllers that derive control error directly from Cross Track Error; a relative measure of vehicular position generally defined as the perpendicular, or shortest, distance between aircraft and path, as illustrated in Figure 2.5a. Although minimising cross-track error is a key objective of the path following task, its direct use as a control error is rarely seen, rather is generally avoided [93] as it can lead to poor performance in the presence of large errors in cross track distance, velocity or heading [47]. In contrast, path following strategies for fixed wing aircraft generally derive from appropriate control of aircraft attitude, akin to the techniques employed by pilots and about which the platforms are inherently designed [86].

Path following based on ‘Good Helmsman’ Behaviour derives from the intuitive actions of a helmsman commanding a ship on a straight path [94], where desired course angle is derived as a function of cross-track error. Modelling desired course as a function of cross-track error expressed as a sigmoid function, allows the vehicle to approach the path at a ‘maximum intercept’ angle from
far away, governed by the sigmoid saturation limit, while controlling the rate of converge closer to the path as a function of the sigmoid slope (Figure 2.5b). Originally posed for UAV tracking by Rysdyk in [92], applications of the technique generally utilise a Serret-Frenet reference frame that allows a continuous measure of cross-track error and course error with respect to the desired path [49, 83].

Vector Fields, also known as ‘Guidance Vector Fields’, are an approach to path following derived from potential fields that generate reference velocity vectors in the area surrounding a path, as illustrated in Figure 2.5c, providing heading and speed commands to a lower level controller to execute a smooth convergence with the desired path [95]. Derived from principles of potential fields and their
use in path planning [96], vector fields for UAV guidance were first proposed by Lawrence in [97], in which a solution was sought to establish and maintain a holding pattern over a stationary target. Although vector fields had previously been applied to obstacle and collision avoidance for UAVs, their application to guidance presented an issue in that the vehicle would be directed ‘into’ the target, while in order to observe a target effectively the aircraft would need to establish a holding pattern. To solve this, Lawrence proposed a solution through Lyapunov functions and a ‘reverse’ application of Lyapunov stability theory to generate vector fields with curl such that the vehicle would enter a loitering pattern, i.e. circular flight path, above the area of interest [97]. These techniques have subsequently been extended to the following of straight paths and arcs [81, 98], racetrack patterns [95, 99], switching of loitering circles [100] and stand-off target tracking for single [101] and multiple UAVs [84, 102, 103].

Creating a Virtual Target that progressively moves or ‘slides’ along the path at a fixed distance in front of the vehicle, as illustrated in Figure 2.5d, is a popular path following strategy for both straight, curved and piecewise linear paths. Implementations of Virtual Targets for Fixed Wing UAV path following vary and include the work of Jung and Tsiotras who propose heading rate control through backstepping of roll angle commands following an approach path generated by vector fields local to a reference point moving along the path [55]; Yoshitani who proposes a Proportional-Integral-Derivative (PID) control scheme that utilises two sliding points termed ‘aim-head’ and ‘feedforward’ to govern directional and curvature control respectively [90]; and augmentation of an $L_1$ adaptive output feedback controller by Kaminer et al. for use with an off-the-shelf autopilot to handle model uncertainty and disturbances [86].

The use of Virtual Targets for Fixed Wing UAV path following has also seen the extension of missile guidance techniques. In the work of Park, Deyst and How, a lateral acceleration controller is developed through principles of Proportional Navigation for improved tracking on curved trajectories [85, 104]. Similar techniques form the basis of lateral control for a UAV forced landing system developed by Eng, Mejias, Walker and Fitzgerald to control the unpowered decent of a UAV during an emergency landing, utilising Proportional Navigation techniques to follow a 3-D Dubins Path [6]. Missile guidance techniques are also utilised by Bruggemann, Ford and Walker in the development of a lateral track controller for a fixed-wing inspection aircraft, where a Precision Guidance law is developed
to control both position and orientation of the vehicle during tracking to ensure infrastructure remains within the predicted field-of-view of body-fixed inspection sensors [22]. Tracking the point at which an infrastructure enters the FOV of a downward facing imaging sensor is proposed by Holt and Beard for road following in [11], posing the entry point as a virtual target for a Proportional Navigational (PN) guidance law. The solution applies the vision-aided PN guidance law developed by Beard et al. for point targets [105] with adjustments to account for steady state error introduced through perceived target acceleration due to bends and turns in the road, resulting in an Augmented PN guidance law.

Real time tracking through vision-based control has seen further applications of virtual targets, with earlier work including the Velocity Ratio controller proposed and demonstrated by Frew et al. for lateral control of a UAV during road following [87, 93]. Developed to exploit the geometric relationship between desired aircraft position and velocity, the control law estimates relative yaw and lateral displacement from imagery to create a proportional feedback controller through turn rate. Another application of Virtual Targets for vision-based road following is presented by Frew in a comparison study of lateral controllers that includes three ‘Aim-Ahead’ controllers all deriving error terms from desired heading, with all three maintaining convergence in the presence of unknown wind [9]. Rathinum et al. likewise present a vision-based road following controller for a fixed-wing UAV, detecting the roads in real-time and applying cubic-spline curve fitting to generate a desired path for the UAV to follow [10, 19, 46]. A tracking controller then attempts to intercept a point along the path defined by a look-ahead distance, commanding yaw rate proportional to the curvature of a 3rd order polynomial generated as a connecting contour for the UAV to join the path.

2.1.2 Field of View Control

In the context of inspection and observation tasks, the ability to track and follow a desired trajectory or path only addresses a portion of the problem, as taking place simultaneously is an underlying data collection process that is directly influenced by platform motion [22, 27]. On the surface, Vision-Based control techniques would appear to inherently address the issue, as feature detection and extraction require the object to remain visible during tracking. This is not necessarily the case however, as many of the developments in Vision-Based control for Fixed
Wing UAVs still define the problem in the 3D task space, posing it as one of minimising physical cross track error with respect to the feature centreline [9, 46, 87, 93], providing no control of object position within the sensor FOV. The impact of manoeuvres using this approach are even highlighted by both Rathinam et al. [46] and Bencatel et al. [91], warning of the potential for features to leave the sensor field-of-view, although neither seeks a solution to address the problem.

Only recently has the objective of controlling aircraft motion in such a way to ensure optimal viewing angle for onboard sensors during data collection become an active area of research. Of principle concern are Bank-to-Turn manoeuvres that directly roll the aircraft about the longitudinal axis, vectoring the main lift force produced by the wings to create a lateral component enabling the vehicle to turn. While an efficient and effective means of altering heading for Fixed Wing platforms, the banking manoeuvre directly rotates downward facing onboard sensors away from the turn and given the aircraft is manoeuvring towards the feature, angles sensors away from the objective [22, 46, 91]. Of research that seeks to address these issues, solutions can be categorised as one of three approaches; altering the planned path to accommodate sensor footprint, reducing or limiting aircraft rotation to avoid features leaving the sensor FOV, and decoupling sensors from aircraft rotation through a gimballed mount.

Decoupling platform motion from onboard inspection sensors through a gimballed mount, allows Sensor Line-of-Sight (LoS) to be controlled independently of aircraft attitude and is a common solution adopted for the surveillance of point targets [106–112]. Utilising a gimballed mount to address the issues of manoeuvres during path following is an approach taken by Holt and Beard for a Bank-to-Turn constrained UAV [11]. Through equal but opposite rotation of the gimbal to that of roll induced by the banking manoeuvre, the inspection sensor is able to remain in a downward facing orientation. This not only reduces the risk of the object leaving the sensor FOV, but allows the aircraft to be modelled as a Skid-to-Turn (STT) vehicle from the perspective of the visual guidance law. Utilising a gimballed mount to point sensors towards the object during tracking is an approach taken by Lee et al. in [113] that allows compensation for offset from the object centreline during tracking that would otherwise see features offset in the sensor FOV. This is achieved through the calculation of line-of-sight angles that account for both offset and bank angle, determined by the turning rate required to traverse the path. The range and rate at which such systems can
operate is however limited, a point highlighted by Lee et al., and if not taken into consideration can still see features leave the sensor FOV.

Adjusting the flight path to take into account the projected line-of-sight of onboard sensors and adopting manoeuvres that avoid unwanted motion, not only reduces the overall compensation required by a gimbal, but can potentially remove the need for one altogether [63]. This is particularly beneficial for platforms with limited payload capabilities [106], and inspection tasks that require multiple sensors that would otherwise be impractical to mount in a single gimbal [25,27]. While relatively new, the concept has seen recent application to fixed-wing UAVs, particularly in the observation of stationary targets. Side-mounted cameras aligned with the aircraft wing, are a popular camera configuration for observing stationary targets and have been utilised by both Gans et al. through path planning and the selection of elliptical orbits [114], and Saunders et al. through a nonlinear image-based controller for real time tracking [115,116].

Extending this to the inspection of periodically spaced points of interest along linear infrastructure (e.g. power poles) is a problem addressed by Bruggemann and Ford [16]. The problem is constrained by the requirement to provide a desired “look-angle” for a downward facing, body-fixed sensor that create specific attitude requirements that are addressed through a precision guidance law. Path planning to capture video footage of an arbitrary set of ground targets is addressed by Ceccarelli et al. in [117], controlling the viewing angle of both a forward facing and side facing imaging sensor through waypoint selection to utilise an existing Autopilot capable of Waypoint Navigation.

In the context of path following, Bruggemann, Ford and Walker propose a guidance law to track linear infrastructure, maintaining desirable relative position and orientation through a precision guidance law derived from missile guidance techniques [22]. This is then complemented with the adaptive selection of aircraft manoeuvres to improve attitude behaviour, utilising a combination of traditional Bank-to-Turn, Constrained BTT (CBTT) limiting maximum bank angle, and Skid-to-Turn manoeuvres. The advantages of STT manoeuvres are also exploited in the work of Holt and Beard [11], for which the vision-based proportional navigation controller is developed upon the dynamic model of a STT vehicle fitted with a downward facing camera. This is then adapted to a BTT constrained UAV emulating STT manoeuvres with a geostabilised gimbaled mount allowing the camera to remain vertical throughout the flight.
Alternatively, the path can be planned from the perspective of the sensor line-of-sight, creating a trajectory that once executed effectively flies the sensor ‘along the path’, as illustrated in Figure 2.6a. This is the approach taken by Jackson et al. in [118], that compares two solutions, the first utilising a spatial sliding mode controller to track a path that accounts for sensor footprint, while the second provides direct control of sensor line-of-sight through a receding-horizon kinodynamic controller that derives control error from current line-of-sight angle and desired camera projection.

![Diagram](image)

(a) Adjusting flight path to fly an inspection sensor ‘along’ a desired path [118]  
(b) Constraining bank angle and AGL so features remain visible during turns [8]

**Figure 2.6:** Field-of-View control examples for downward facing, body-fixed sensors where emphasis is placed on controlling the line-of-sight vector of the sensor (a) and projected sensor footprint (b).

Limiting manoeuvres to avoid features leaving the sensor field-of-view during vision-based road tracking is a problem addressed by Egbert in [8]. Given sensor field-of-view angle and relative position, constraints for both roll angle and height about ground (AGL) are calculated to ensure features remain visible for a BTT constrained UAV, resulting in a footprint projection as shown in 2.6b. This is later extended in [119] to address the position at which the feature is viewed in the camera footprint during the turn by initiating the turn earlier given a priori knowledge of the object location. While these techniques provide effective means of ensuring features remain visible during tracking, the feature is not necessarily centred during tracking, nor are the effects of rapid body rotations considered that can induce motion blur in captured data [24, 27, 120].
2.1.3 Discussion

A key component in the autonomous operation of fixed-wing UAVs is the ability to follow a desired path, with applications ranging from navigation between two points avoiding obstacles, to the observation of ground based features and guidance during emergency landing. Subsequently a wide range of techniques and approaches have emerged to address path following, although on a whole can be broadly divided between solutions that assume knowledge of infrastructure position and those that sense the object in real-time.

Inherently, pure position-based control techniques that rely on a priori knowledge of infrastructure location will face reduced tracking performance in the presence of positioning errors. The subsequent impact on data collection is unavoidable given the direct link between sensor FOV and relative positioning of aircraft and feature. While measures to avoid positioning errors are feasible in controlled environments, infrastructure inspection is likely to encounter a mixture of new and old data, geolocated through a range of systems with varying datums, that were never intended as precision navigation aids.

Although small scale positioning errors may only result in a feature offset within the FOV of onboard inspection sensors, large scale errors have the potential to cause infrastructure to leave the FOV of sensors, particularly those with narrow fields of view. Assuming a position-based controller can guide the UAV within detection range, real-time sensing offers the ability to develop closed loop control to not only address positioning errors, but account for uncertainties and disturbances. While existing vision-based control techniques would appear to inherently address the issue, the majority of work in the area propose a solution through position-based control techniques.

In doing so, the problem is posed in the 3D task space, where relative position and orientation are estimated to develop a controller that minimises physical cross-track error with respect to the feature centreline, offering no control of feature position within the sensor FOV, and thus captured data. Ideally the position and orientation of the aircraft would be controlled to ensure the view of downward facing inspection sensors is optimal for data collection.

An exception to this is the work of Holt and Beard in [11], where steady state error in the image plane is used to correct tracking errors of the PN guidance law. The control task still focuses on converging over the feature, as opposed
to achieving a desired view, which is reflected in the control design; defining a virtual target at the point where the feature enters the FOV, as illustrated in the sequence of Figure 2.7a. While the technique ensures features remain visible during tracking, it can be seen that it provides no control over the overall position or orientation of the feature during tracking.

![Simulation Sequence of Control Algorithm Introducing Steady State Error](image1)

![Experimental Results showing Image Error during Road Following](image2)

**Figure 2.7:** Results from the Augmented Proportional Navigation Guidance law proposed by Holt and Beard for Road Following [11]. Technique poses the point at which a feature enters the FOV as a Virtual Target, as indicated by a red dot.

In addition to controlling the position of an object within the sensor FOV, it is also desirable to minimise sensor motion to avoid corrupting captured data with motion blur that, to date, is still an open area of research. An example of the problem is evident from results presented by Holt and Beard, shown in Figure 2.7b, where considerable motion of the feature within the image plane is seen during tracking. Developing a controller that accounts for position, orientation and motion of features from the perspective of sensor FOV would appear to be possible from vision-based control and is thus explored in greater detail.
2.2 Vision Based Control

The use of vision in robotics for guidance and navigation is a well established field of research, one that has seen continuous development since the 1970s [121]. While many of the original techniques took upon a ‘look’ then ‘move’ approach, where visual information provided cues for subsequent action from the robot, later development saw the inclusion of visual information in closed loop feedback, allowing control of robot motion through a technique now known as Visual Servoing. Generally recognised as being introduced by Hill and Park in 1979 [122, 123], the technique became widely popular for the control of robotic manipulators [124–127].

In general, visual servo techniques can be broadly classified as either Position-Based, Image-Based, or a combination of the two known as hybrid techniques [123]. Position Based Visual Servoing (PBVS), or 3-D Visual Servoing, utilises features extracted from an image to reconstruct target pose with respect to the camera given a known geometric model of the target, thus providing an estimate of system states. A simple example is depicted in Figure 2.8, illustrating the application of PBVS to a 1D positioning task controlling a camera relative to a fixed object. For a PBVS solution, distance from the object, $x_c$, would be estimated from a measurement of extracted feature diameter, $i_c$, given the known diameter of the object, $d$, and camera model. The resulting controller would then take a simple error signal ($e = x_c - x_d$) driving position error to zero. While allowing for straightforward control design, the technique is inherently sensitive to camera calibration and accuracy of the target model [128].

Image Based Visual Servoing (IBVS), or 2-D Visual Servoing, on the other hand, avoids pose estimation by controlling the task directly from the image plane. This is achieved by manoeuvring the vehicle such that the detected features are observed in a final goal pose. In the example of Figure 2.8, an IBVS control solution could be sought recognising that the feature will appear as shown in Figure 2.8d once the positioning goal is achieved. A simple relationship between camera motion (forwards/backwards) can then be linked to change in feature diameter as measured in the image plane, $i_c$, to create a simple control law that drives $i_c$ to $i_d$. While overcoming camera calibration and target modelling issues, control design can become complicated as degrees of freedom increase and vehicles dynamics are introduced [129].
Literature Review

Figure 2.8: Visual Servoing example for relative positioning of a cart with respect to a fixed object. PBVS control would derive error, \( e = x_d - x_c \), estimating \( x_c \) from \( i_c \) given the camera model and diameter of object, \( d \), are known. IBVS derives error, \( e = i_d - i_c \), requiring no conversion or estimates, although requires a relationship between cart movement and image diameter of the object to be established.

Placement of the imaging sensor within the task space further divides visual servo controllers into categories of ‘eye-in-hand’ and ‘eye-to-hand’ configurations. Visual servo controllers utilising an ‘eye-in-hand’, or ‘hand-eye’, configuration see the imaging sensor mounted on the vehicle or end effector, allowing the controller to observe vehicular motion relative to a set of known objects within the task space \([123, 125, 126]\). Control systems on the other hand designated as ‘eye-to-hand’ \([130, 131]\), ‘static-eye’ \([132]\) or ‘stand-alone’ \([127]\) configurations, refer to systems that utilise a camera to observe motion of the vehicle or end effector, that is either fixed in the workspace or itself attached to a moving platform (e.g. fixed to another vehicle for cooperative control).

Control design can take advantage of either configuration and be used in conjunction with both position-based or image-based techniques, although ‘eye-in-hand’ configurations are found to be far more common among UAV applications, particularly fixed-wing UAVs, as is discussed in proceeding sections.
2.2 Vision Based Control

2.2.1 Position-Based Control

The convenient separation of pose estimation and control offered by Position-Based Visual Servoing (PBVS) has seen the technique applied to a variety of applications for visual control of UAVs.

For UAVs capable of hovering and near-hover flight, relative positioning with respect to both fixed and moving targets has been a popular application of visual servoing [133–135]. In particular, PBVS solutions enabling relative alignment of a helicopter with respect to a marker during autonomous landing has seen considerable attention [136–140], utilising a distinctive target such as a helipad symbol (H) that is both readily detected by a downward facing sensor and well structured for pose estimation.

Autonomous landing of fixed-wing UAVs using PBVS have also been proposed, including pose estimation from point features detected on a runway by Dickmanns and Schell [141], detection of Approach Lighting System (ALS) beacons by Chatterji et al. [142], through to the PBVS solution of Gonçalves et al. using dense visual tracking for increased accuracy and robustness under arbitrary lighting conditions [143]. Relative positioning for eye-to-hand systems, utilising a camera to observe motion of the UAV, have also attracted the application of PBVS including the use of both single [144] and multiple ground cameras [145,146], as well as combining the view of an onboard camera [147–149].

Cooperative control between UAVs and Unmanned Ground Vehicles (UGV) has been another popular application of PBVS, where relative positioning of the air vehicle is sought with respect to the ground vehicle. Solutions for both eye-to-hand and eye-in-hand systems have been proposed, with the former reducing weight and complexity of the UAV [150], and the latter allowing the UAV to operate independently of the UGV [151]. Similar techniques have also been extended to the autonomous landing of helicopter and quadrotor UAVs on moving vehicles [152–155].

Autonomous aerial refuelling (AAR) has been a topic of interest for many years, with applications of visual servoing dating back to the 1980s and controllable booms [156, 157]. Later work has considered visual servoing solutions to address relative positioning between Refuelling Tanker and UAVs and include solutions for both the US Air Force “Flying-Boom” and US Navy “Probe-Drogue” refuelling systems. For flying-boom systems, the aircraft must maintain position
within a 3D window relative to the tanker from which position the controllable boom is guided into position and has seen PBVS solutions for UAVs fitted with forward facing cameras detecting both markers [158] and prominent features of the tanker [159,160]. Probe-Drogue refuelling systems remove the need for a controllable boom, utilising an aerodynamically stable drogue trailing the tanker, although the system places increased demand on the receiving aircraft control system, requiring centimeter level accuracy that only a few systems, including PBVS, can offer [161–164].

As was previously discussed in Section 2.1.1, path following for fixed-wing UAVs has seen a number of vision-based control solutions. A number of these employ PBVS techniques and include the work of Frew et al. [9,87,93], estimating cross track error in the development of the Aim-Ahead controllers; Rathinum et al. [10, 19, 46], generating desired paths through curve fitting of the detected feature; and Holt and Beard [11] controlling the point where infrastructure enters the FOV as a point target for a PN guidance law.

### 2.2.2 Image-Based Control

Given the goal of a control task can be described either through the desired view of objects from a camera fitted to the vehicle under control, or through a final view of the vehicle as observed by an imaging system capturing motion of the vehicle, then an alternative approach can be taken to the visual servoing task through the application of Image Based Visual Servoing (IBVS). By defining the control task directly within the image plane, the controller is inherently robust to camera calibration and alleviates the requirement for a 3D model of the object [123], rather, can utilise any combination of 2D features including points, lines, circles, and image moments [165].

In the case of robotic manipulators, for which many of the classic techniques were developed, IBVS control designs generally fulfil the roll of an outer loop controller, providing velocity commands to an inner loop controller [125]. Control is then achieved through the construction of a Jacobian matrix relating rate of change of feature parameters to the rate of change of camera pose, also known as the image Jacobian or interaction matrix [124], for which a velocity controller can be developed through inversion of the interaction matrix [126]. In the context of fully dynamic underactuated systems however, commanding desired velocities
is no longer suitable as the vehicle dynamics limit the ability to generate direct accelerations, requiring more involved control developments [129].

Both the benefits and challenges of applying IBVS control to UAVs has subsequently seen an increase in interest over recent years, with many of the applications that attracted PBVS solutions reflected in applications of IBVS. Relative positioning with respect to fixed targets is again a popular application for those vehicles capable of hover or near hover flight, including helicopters [5, 166–169], quadrotors [170–174] and blimps [175, 176]. Observing stationary targets from fixed-wing UAVs requires an alternate approach to ensure forward velocity is maintained for the generation of lift, typically accomplished through the execution of circular or elliptical orbits that have likewise seen the application of IBVS for real-time tracking [109,115,116,177]. Automated landing of fixed-wing aircraft has also been a popular application of IBVS control, utilising a desired view of runway features to achieve control during each phase of landing including alignment, glideslope and final flare [143,178–182].

In the context of path following, the use of IBVS control has been a popular application to both blimps and quadrotors. In the earlier work of Silveira et al. addressing vision-based road following for blimps, the control task is posed as one of following three parallel lines as would be extracted from the centreline and edges of a road [183,184]. The three lines provide sufficient information to construct an image Jacobian of full rank, utilising the interaction matrix development of Espiau, Chaumette and Rives for line features [165], that allows for the inversion of the interaction matrix and the design of a velocity based control law. This work is later extended by Silveira et al. to address vehicle dynamics for both lateral control [12] and full control of the blimp through IBVS [185], augmenting the linearised state space system model with the image Jacobian such that the states of the system include the vector of visual signals.

In many instances however, extracting sufficient features to form an image Jacobian of full rank will prove impractical, including rivers, pipelines and rural roads that offer two features from borders, and rural power lines and smaller pipelines that limit feature extraction to a single line feature. This forms the motivation for the work of Silveira et al. in [186], where only two line features are assumed available for control. With only two features available for visual servo control, two degrees of freedom remain unconstrained, a problem that is solved through the use of complementary sensors in the form of an airspeed sensor and
rotational sensor. A similar problem is addressed by Rives and Azinheira in [187] for road following, where feature extraction of the object is supplemented with detection of the horizon and vanishing point of features through a forward facing imaging sensor. This not only provides additional features, but offers a convenient decoupling of rotation that is natural to airship control.

Mahony and Hamel propose a general IBVS control solution for vehicles capable of hover and near-hover (quasi-stationary) flight, tracking linear features through a downward facing camera [188]. Control of the vehicle is modelled upon a single translational force combining thrust, lift, gravity and drag, that can be vectored through torque control available about the three orthogonal body axes, capturing the general arrangement of vertical take off and landing (VTOL) platforms (e.g. helicopter, quadrotor). Line features in this instance are expressed in the form of Euclidean Plücker coordinates that allow the decoupling of line tracking and velocity regulation, while preserving passivity-like properties of rigid-body dynamics that the proposed design exploits to derive a control Lyapunov function using backstepping techniques. An extension of the work is presented by Mahony and Hamel in [129], including the simulation of an X4 flyer quadrotor under a number of conditions including realistic frame rates and high frequency noise.

In the context of fixed-wing UAVs, the application of IBVS to path following is yet to be explored, although can draw up techniques developed for automated landing and runway alignment for which IBVS solutions have been proposed. Bourquardez and Chaumette propose an IBVS control solution for runway alignment of a fixed wing UAV, utilising a forward facing camera to detect runway centreline, left and right borders, horizon and vanishing point [178]. Motion of each feature is linked to aircraft motion through point and line interaction matrices formed using the techniques of Espiau et al. [165] from which control laws are developed upon the decoupled equations of motion for lateral and longitudinal motion. Lateral control is achieved through a number of simplifications linking aileron deflection to roll rate, roll rate to heading angle and heading angle to lateral position to form a direct control law from image measurements. Longitudinal control is then sought through an LQR control design, estimating states through the previously developed visual feature links.

This work is then extended from runway alignment to autonomous landing by Azinheira and Rives in [189], once again utilising a forward facing imaging sensor to detect left and right borders of the runway, the horizon line and vanishing point.
of border features. The authors apply an earlier development of IBVS control that addressed road following for an airship [187], linearising both the equations of motion and interaction matrix about a horizontal, straight and level trimmed flight condition, for which the image Jacobian is then incorporated in the output equation of the state space system model. This allows the design of an LQR controller, for which sliding gain is applied to account for variations in the image Jacobian as the vehicle nears touchdown, while an integral term is incorporated to account for steady state error introduced through wind disturbances.

2.2.3 Discussion

Although Position-Based Visual Servoing techniques provide a convenient separation of state estimation and control design that have lead to numerous developments in UAV applications, it is important to consider that the technique remains inherently sensitive to camera calibration and modelling errors [129]. While practical measures can be taken in controlled environments to limit such errors, the outdoor environment in which inspection UAVs will operate is highly unstructured and impractical to model.

Furthermore, and of particular concern to the application of inspection, is the lack of control offered by PBVS to the view of features during the control task, as the control task remains focused on achieving goals set within the 3D task space. While the impact of minor errors may only lead to features offset in the sensor FOV, there is an ever present issue of features potentially leaving the sensor FOV given no other measures are put in place [190]. Adopting alternate manoeuvres may reduce the risk of features leaving the sensor field of view, e.g. Skid-to-Turn manoeuvres, however their use in coordination with PBVS control still provides no means of controlling feature position or motion.

Image Based Visual Servoing on the other hand allows the problem to be posed directly within the image plane. In the context of inspection, this has the potential to pose the control task directly from the perspective of data collection, controlling the view of infrastructure from its detected features. While path following applications of IBVS control have been presented for UAVs capable of hover and near-hover flight, the application to fixed-wing path following is unexplored. The development of IBVS control for runway alignment and autonomous landing provide useful insight into possible solutions.
To date though these have been posed through forward facing sensors that detect both features, their vanishing point and the horizon (Figure 2.9). The influence of roll induced through Bank-to-Turn manoeuvres is also less pronounced in forward facing sensors, while the same manoeuvres can have dramatic effects on downward facing imaging sensors that would be used for inspection purposes, and require special consideration in control development.

Figure 2.9: IBVS solutions for Automated Landing of fixed-wing UAVs. While sharing similar goals to inspection, forward facing sensors provide view of horizon and vanishing point, with less risk of manoeuvres compared to downward facing sensors.

2.3 Visual Detection

Successful vision-based control relies on the accurate and reliable detection and extraction of features, specific to the given application. The following of roads by fixed-wing UAVs has been a popular application that has led to a number of solutions for real-time road detection from low-altitude aerial imagery. Identifying the road from lane markings is proposed by Silveira et al. in [183] through the use of Mathematical Morphological filters, while Frew et al. detect lane markings having first identified candidate road pixels through a Bayesian RGB pixel classifier [93].

Detecting the road given the uniform colour and contrast between surrounding areas (Figure 2.10) has also seen other solutions including Kim through a semi-supervised learning algorithm generating a cross-section profile of a sample image for matching with the horizontal scan lines of target images [191]; Egbert
through a statistical classification method in the HSV colour space [8]; and Lin and Saripalli detecting rural roads through a histogram based adaptive threshold algorithm [192].

![Feature Extraction](image)

**Figure 2.10:** Detection of Road from downward facing camera, Egbert [8].

Aerial inspection of power transmission networks is another popular application that has led to the development of detection and extraction of power lines from low-altitude aerial imagery (Figure 2.11a-b). Given the conductor appears straight from an overhead vantage, the problem can utilise any number of well established straight line extraction techniques that have arisen in the last 40 years [193], with many adopting the popular Hough transform. First filed in a patent application by Hough in 1960 [194], the technique was later adapted for detecting lines and curves in images by Duda and Hart in 1972 [195] and applies a parameter space mapping of pixels identified by an edge detector to identify the location and orientation of features (e.g. lines, circles).

Reducing noise in preprocessed edge images is particularly important for systems detecting features against complex and cluttered backgrounds that has led to a number of variations in edge detection methods used for Hough transform based power line detection. These include the work of Li et al. and the use of a Pulse Coupled Neural filter in producing an edge map [196], a modified Marr-Hildreth edge detector by Tong et al. combined with Morphological analysis [197], and the use of Dissimilarity textural differentiation and a Nearest Neighborhood clustering algorithm by Wu et al. [198]. Yan et al. utilise the closely related Radon transform in place of the Hough transform, utilising an extended line mask to reduce noise and a Kalman filter to track broken line segments [199]. Tracking
power lines between successive frames can also improve detection in cluttered environments, as well as reduce detection times, and is proposed for power line detection by both Zhang et al. utilising a Kalman filter to track the line in the Hough parameter space [200], and by Candamo et al. combining motion estimation and edge detection followed by a windowed Hough transform [201].

For certain systems, detecting the power line conductor may prove impractical as the combination of altitude, sensor field-of-view and resolution may result in insufficient spatial resolution to resolve the conductor. In these instances an alternate solution may be sought by inferring the location of the conductor from detection of the supporting power poles, as is proposed by Sun et al. in [203], for which a number of power pole [202, 204, 205] and transmission pylon [206] detection algorithms can be utilised (Figure 2.11c-d). The observation and monitoring of natural features has also attracted the use of UAVs that has led to vision based detection algorithms for both rivers [19] and coastlines [207].
2.4 Summary

A major step towards automated infrastructure inspection is the automated collection of data, that in the context of UAVs can be considered an application of simultaneous path following and attitude control, enabling onboard inspection sensors an optimal view of infrastructure. This varies from the common strategy of path following employed by aerial vehicles that poses the problem as one of flying directly over the path, minimising cross-track error with respect to the object centreline. Applying such an approach to aerial vehicles that manoeuvre utilising body rotations, neglects variation that arises between the flown path and projected line-of-sight of onboard sensors, resulting in data capture that no longer adheres to the intended inspection route.

While research has sought to address the issue at a path planning level, the techniques rely on a priori knowledge of infrastructure location to execute the flight path. It is then trusted that onboard inspection sensors capture an adequate view of the feature as the technique provides no form of feedback with respect to infrastructure and it’s position within the sensor FOV. Detecting and tracking infrastructure in real-time can offer reassurance that data collection has accurately captured the object under inspection, and provides the ability to account for any positioning or alignment errors that may be present that would otherwise see features offset or leave the FOV of sensors. Although a number of vision-based path following algorithms have been developed for fixed-wing UAVs, control is still posed in the 3D task space, utilising visual information to provide relative positioning information to a controller minimising cross track error, providing no control of feature position with the sensor FOV.

Image Based Visual Servoing would appear to offer a direct means of developing the control task from the perspective of inspection, offering aircraft control such that a desired view of features is observed from the body-fixed sensor. Numerous image based detection methods have been developed for the extraction of common infrastructure from aerial footage, including roads and power lines, providing position and orientation of the feature as detected in the image plane that could be utilised by an IBVS controller. By controlling the field of view of one sensor, simultaneous path following and field of view control is inherently achieved for all sensors with equal alignment, thus addressing the problem for platforms equipped with multiple inspection sensors. While applications of IBVS
have been proposed for a number of UAV tasks, their application to path following has to date been restricted to vehicles capable of hover and near-hover flight. Closely related work includes that of autonomous landing for fixed-wing UAVs, although of those solutions presented, the problem is posed through a forward facing sensor as opposed to a downward facing sensor that would be used for inspection. This not only reduces the number of features available for visual servoing, as the horizon and vanishing point of features are no longer in view, but must deal with the ill effects of manoeuvres that directly affect downward facing sensors.

Combining the benefits of both IBVS control and alternate manoeuvres would appear to offer a new and novel means of addressing the issues that surround infrastructure inspection for fixed-wing UAVs equipped with downward facing sensors. There also appears to be an unexplored avenue of utilising manoeuvres that not only avoid unwanted motion, but assist data collection, allowing sensors to be angled towards the object during tracking. Reducing unwanted motion of inspection sensors during tracking is another open research topic that has received little to no recognition in the field of UAV tracking, such that a solution in visual servoing would be novel in its own right. From this literature review, it can be seen that the application of infrastructure inspection to fixed-wing UAVs poses a number of unanswered questions that can draw upon relevant fields of research to develop novel and beneficial solutions to real world problems.
This chapter investigates direct control of data capture for fixed-wing UAVs operating body-fixed downward-facing sensors through the use of visual servoing. As was reviewed in Section 2.1.2, issues surrounding simultaneous path following and data collection have only recently been considered, with those solutions focusing on the problem from the perspective of path planning.

Addressing the problem at such a level places considerable reliance on accurate a priori knowledge of infrastructure location that is not always available at the precision required for such guidance tasks. Detecting and tracking the feature in real-time provides a means of ensuring data collection is preserved throughout the inspection task, even in the presence of positioning errors and outside disturbances. While Position-Based Visual Servoing (PBVS) has been a popular solution in the visual control of UAVs, as discussed in Section 2.2.1, and would provide a solution to real-time tracking, the technique provides no control over feature position within the sensor field-of-view.

Image-Based Visual Servoing (IBVS) on the other hand allows the task to be developed from the perspective of achieving a goal pose of features as detected in the image plane. In the context of inspection, this would allow aircraft control
to focus on achieving an optimal view of infrastructure from the perspective of onboard inspection sensors. While IBVS control through a forward facing sensor has been proposed for runway alignment of fixed-wing UAVs during autonomous landing \([178, 189]\), the application of IBVS to simultaneous path following and data collection through a downward facing sensor has yet to be presented.

This chapter presents a novel solution to the simultaneous tracking and data collection problem through an IBVS control design employing wings-level Skid-to-Turn manoeuvres to maintain a desired view of features. The chapter begins with an overview of current issues and the concept of the proposed solution, followed by the derivation of the control law. Performance of the proposed controller is then assessed against that of a position-based visual controller utilising standard bank-to-turn manoeuvres over a range of practical operating conditions.

### 3.1 Problem Formulation

A major contributing factor to the issues surrounding data collection is the difference between the path flown over ground to that of the projected line-of-sight of onboard sensors. This occurs when the aircraft is rotated from a straight and level flight condition that occurs frequently as the UAV alters course to compensate for disturbances and manage the transition between segments. Through the use of conventional techniques, the aircraft is commanded to rotate about the longitudinal axis in order to perform what is commonly known as a Bank-to-Turn (BTT) manoeuvre.

#### 3.1.1 Bank-to-Turn Manoeuvres

Performing a Bank-to-Turn manoeuvre begins by rolling the aircraft about the longitudinal axis initiated through the deflection of ailerons until a desired angle of bank, \(\phi\), is reached. At this point the resultant lift vector produces a horizontal component of force, as shown in Figure 3.1, that leads to centripetal acceleration and thus a change in aircraft heading. Given the aircraft is flying at constant velocity, \(V_0\), and performs a level turn, then radius of the turn, \(R\), can be expressed as follows,

\[
R = \frac{V_0^2}{g \tan \phi}
\]
3.1 Problem Formulation

where angular rate of the turn will correspond to change in course, \( \dot{\chi} \), that can be expressed as,

\[
\dot{\chi} = \frac{g}{V_0} \tan \phi
\]  

(3.2)

Whilst an effective means of altering the course of a fixed wing UAV, the manoeuvre equally sees onboard sensors rotated away from their original downward facing orientation. When repositioning and transitioning between segments, this sees sensors angled away from the direction of turn and consequently the inspection task. The rate at which the aircraft transitions from wings level flight to banked flight can also have an impact on data collection, as rapid rotation of the sensor results in panning motion that has the potential to induce motion blur in captured data. These issues only compound as the UAV attempts greater changes in heading, increasing the required angle of bank, pointing sensors further away from the inspection task, which in the extreme can see features leave the sensor FOV altogether. While limits on bank angle and roll rate may reduce these effects, they equally restrict the rate at which the aircraft can then turn, thus causing the UAV to fly further from the intended path.

Adopting alternate manoeuvres that avoid bank has been proposed for position based controllers [11, 22] and one that would appear to equally assist vision based control. One such manoeuvre that allows the aircraft to remain level during a turn is the Skid-to-Turn manoeuvre.

**Figure 3.1:** Level Bank-to-Turn Manoeuvre
3.1.2 Skid-to-Turn Manoeuvres

As opposed to the BTT manoeuvre that requires the aircraft to bank, a Skid-to-Turn (STT) manoeuvre utilises rudder to initiate a sideslip angle, $\beta$, between the longitudinal axis and relative airflow as illustrated in Figure 3.2. The resultant thrust vector produces a component of force perpendicular to the relative airflow, while additional aerodynamic forces are generated by the now exposed fuselage and vertical stabiliser. The result is a centripetal force relative to sideslip angle, $F_s(\beta)$, in the horizontal plane that allows the aircraft to alter heading. Assuming ailerons are operated to maintain wings level flight ($\phi, \dot{\phi} = 0$) while the UAV flies a level flight path ($\gamma = 0$) at constant velocity, $V_0$, then the resulting change in course can be expressed as,

$$\dot{\chi} = -\frac{F_s(\beta)}{V_0 m}$$  (3.3)

where the change in course is negative relative to a positive angle of sideslip.

Positive sideslip is defined by lateral velocity, $v$, acting along the body axis $oy_b$, resulting in the incidence of the freestream airflow ($V_\infty$) upon the starboard
side of the fuselage, as illustrated in Figure 3.2. As a result of directional stability that is inherent in the design of conventional airframes, entering a sideslip and maintaining the angle during flight requires the continued application of rudder such that $\beta \propto \delta_r$. Common convention is adopted, with positive deflection of the rudder defined with the trailing edge deflected towards the port side wing \[208-210\], producing positive sideslip. Once an angle of sideslip is established the incidence of airflow on the vertical stabiliser and variation in lift generated by each wing induces a rolling moment that can be overcome with deflection of ailerons allowing the manoeuvre to be performed with wings level attitude.

As the manoeuvre can isolate rotation of the aircraft about the yaw axis, on-board sensors are able to maintain a downward facing orientation whilst altering course. Skid-to-Turn manoeuvres do however have their disadvantages; hence their limited use in the day-to-day flight of larger manned aircraft. First and foremost is the relatively small amount of turning force the manoeuvre can produce relative to the BTT manoeuvre. As the platform is generally designed with directional stability in mind, the ability to enter and hold a sideslip configuration is limited by the control power of the rudder to overcome the restoring moment generated by the vertical stabiliser.

Having established a sideslip, the fuselage and vertical stabiliser are then exposed to the relative airflow that increases drag, reducing efficiency, while lateral acceleration experienced by onboard passengers can lead to discomfort and motion sickness. For these reasons the STT manoeuvre is generally reserved for special flight conditions, including cross wind landing and aerobatics \[211, \text{ch. 8}\]. Passenger comfort is of course no longer relevant when considering automated inspection, while reduced efficiency during transitions would appear a small compromise given the potential benefit the manoeuvre may offer to data collection.

While the STT manoeuvre has been proposed as an alternative to BTT for position based controllers to avoid unwanted sensor motion \[11,22\], the potential to improve vision-based control for real-time tracking has yet to be explored. As discussed in Section 2.2.3, Image Based Visual Servoing (IBVS) offers a solution to control features directly from the perspective of an imaging sensor that could potentially be extended to the task of inspection. Combining the advantages of STT with IBVS posed through a downward facing sensor has yet to be investigated and would appear to offer a solution to simultaneous tracking and data collection for fixed-wing UAVs and is thus considered in the following section.
3.1.3 Visual Control Design

Design of an IBVS controller begins with the selection of suitable features for control. Figure 3.3a shows an example of aerial imagery obtained from a UAV flying at approximately 30 m (100 ft) AGL over a set of three phase power lines. Although in the example a total of three line features could be extracted, the aim of the research is to develop a generic tracking solution that could be applied to any number of locally linear infrastructure inspection tasks including roads, pipelines and rural power lines. To achieve this, the detected feature is represented as a single line feature, as shown in Figure 3.3b, defined by Sensor Track Error, $T_e$, and Observed Line Angle, $\Theta_{\text{obs}}$.

Ideally, infrastructure is to remain centred in the FOV of sensors for the duration of the inspection process. From a control perspective, this can be seen as minimising Sensor Track Error, $T_e$, while maintaining a Course Over Ground, $\chi$, equal to the feature's orientation with respect to Earth, $\chi_f$, where each of these terms is depicted in Figure 3.4a. It should be noted that the aircraft does not necessarily fly a path directly over the feature to achieve this, as steady state pitch, roll and yaw will angle the line-of-sight of body-fixed sensors away from vertical, requiring the aircraft to fly off centre for the feature to appear centred in the sensor FOV. Under ideal conditions of no wind, and the inspection sensor aligned with the body axis, then the feature can be expected to appear vertically through the image plane during steady state tracking as Aircraft Heading, $\Psi$, 

**Figure 3.3**: Representation of linear infrastructure as a single line feature defined by Sensor Track Error, $T_e$, and Observed Line Angle, $\Theta_{\text{obs}}$ that can be applied to many variants of locally linear infrastructure including power lines, roads and pipelines.
will reflect Course Over Ground, leading to $\Theta_{\text{obs}} = 0^\circ$. It is important to note however that in the presence of wind, Aircraft Heading and Course Over Ground no longer align such that during steady state tracking Observed Line Angle can be expected to vary as a function of the wind vector and aircraft velocity.

Considering initially the ideal case of no wind, the goal of the IBVS controller can be seen as one of centring the feature with vertical alignment, driving both $T_e$ and $\Theta_{\text{obs}}$ to zero. In addition to achieving a desired view, this research also considers controlling motion of the feature, principally to limit any motion that may lead to motion blur. For this reason, control is developed around the concept of a Desired Line Angle, $\Theta_d$; an angle that varies as a function of $T_e$ in order to guide the aircraft on a trajectory that recentres the feature in a smooth transition.

![Diagram](image)

(a) Birdseye view of a UAV on an undesirable course with respect to feature (left), and subsequent view of feature as would be captured by imaging sensor (right).

(b) Desired course at equal $T_e$ that will see the feature move towards image centre, and subsequent view of desired feature (black) versus current feature view (grey).

**Figure 3.4:** Deriving Desired Line Angle, $\Theta_d$, given current Sensor Track Error, $T_e$, to guide UAV on a smooth path to convergence. Control error, $\Theta_e$, is then derived as a function of Observed Line Angle, $\Theta_{\text{obs}}$, and $\Theta_d$. Assumes no wind and zero sideslip.
This not only reduces feature motion, but allows the rate at which the feature approaches the centre to be controlled, in turn providing a means to limit overshoot. To illustrate the concept, Figure 3.4 compares two scenarios, both positioned in such a way to observe equal $T_e$, although at different headings that see one fly away from the feature (a), the other towards (b). In both examples, a birdseye view of the scenario is provided left, while a simulated image as would be captured by the sensor is depicted right.

The concept builds upon the ability to infer relative position and orientation from captured imagery alone. For instance, in the scenario depicted in Figure 3.4a, the feature appears in the left side of the frame, implying the aircraft is to the right of the feature, while the line feature angles away from the image vertical, inferring the aircraft is flying away from the feature. A similar observation can then be made between a desired flight path and a desired view of features, as depicted in Figure 3.4b, where orientation of the feature is seen to reflect Desired Course, $\chi_d$, in the form of a Desired Line Angle, $\Theta_d$.

Control of the UAV can then be sought by minimising the control error, $\Theta_e$, observed between Observed Line Angle and Desired Line Angle. Figure 3.5a shows the relationship between Sensor Track Error and Desired Course having extended the concept over the full range of a desired trajectory, Figure 3.5b.

\[
\chi_f \pm \Delta \chi_{\text{max}}
\]

\[
\chi_f - \Delta \chi_{\text{max}}
\]

\[
\chi_f + \Delta \chi_{\text{max}}
\]

\[
T_e
\]

(a) Desired Course versus Sensor Track Error

(b) Desired Trajectory

Figure 3.5: Relationship between Sensor Track Error and Desired Course (left) that produces the desired trajectory (right) that will lead to smooth convergence of the feature to the image centre.
3.1 Problem Formulation

It should be noted at this point that the sign of $T_e$ reflects the line’s position in the left and right halves of the image plane respectively, with the position of the line determined by the angle $\angle T_e$, as shown in Figure 3.4a, with right ($+T_e$) defined between 0 and $\pi$ and left ($-T_e$) between $\pi$ and $2\pi$. Line Orientation, including Observed Line Angle and Desired Line Angle, is then measured clockwise with respect to the image vertical.

Mathematically, the relationship seen between Sensor Track Error and Desired Course can be described by a sigmoid function in the form,

$$\chi_d(T_e) = \Delta \chi_{\text{max}} \left( 1 - 2 \left( 1 + e^{\frac{T_e}{k_s}} \right)^{-1} \right) + \chi_f$$

(3.4)

where $k_s$ determines the slope of the function as it transitions from the maximum approach angle, $\Delta \chi_{\text{max}}$, and thus determines the rate at which the aircraft approaches the feature and subsequently controls motion during data capture.

A key concept in the example of Figure 3.4 infers Desired Course as a function of feature pose as detected by a downward facing imaging sensor aligned with the longitudinal axis of the aircraft. Not considered in the example however are the effects of wind and sideslip that result in Course Over Ground no longer aligning with Aircraft Heading. With the introduction of sideslip, $\beta$, the longitudinal axis of the aircraft is angled away from the relative airflow such that Aircraft Heading no longer aligns with the Aircraft Velocity Vector, as illustrated in Figure 3.2.

![Figure 3.6: Observed Line Angle as a result of sideslip, $\beta$, and the effects of constant wind, $V_w$, that introduces an angle, $\theta_w$, between velocity, $V_0$, and Course Over Ground.](image)
In the presence of wind, relative velocity between the ground and airmass further alters the angle between heading and course, introducing an angle between the Aircraft Velocity Vector and Course Over Ground. The resultant course angle can then be expressed as,

\[ \chi = \Psi + \beta + \theta_w \]  

where \( \theta_w \) is the angle formed between the Aircraft Velocity Vector, \( \mathbf{V}_0 \), and Wind Vector, \( \mathbf{V}_w \). The impact of these angles on tracking and the subsequent view of features is illustrated in Figure 3.6, where the relationship between course and Observed Line Angle can be expressed as,

\[ \Theta_{\text{obs}}(\chi) = \chi_f + \beta + \theta_w - \chi \]  

\( \Theta_{\text{obs}}(\chi) \) can then be formulated as a function of Desired Course as was derived in (3.4) resulting in,

\[ \Theta_d(T_e) = \Theta_{\text{obs}}(\chi_d(T_e)) = \Delta \chi_{\text{max}} \left( 2 \left( 1 + e^{\frac{T_e}{T_c}} \right)^{-1} - 1 \right) + \beta + \theta_w \]  

Unfortunately, neither wind nor the angle of sideslip can be readily observed from imagery. This presents an issue in the design of the IBVS controller, although can be alleviated by recognizing the conditions under which the terms are introduced. With respect to sideslip, the predominant factor will be the controller itself initiating STT manoeuvres through rudder. It can therefore be expected that sideslip will approach zero as the controller nears steady state tracking conditions, while the maximum angle of sideslip can be regulated through limits imposed on control surface deflections. Given the angle of sideslip would be assumed small and to approach zero during steady state tracking, the angle is neglected from the control law; an assumption that is later shown to have little effect on overall performance in Section 3.4.

Wind on the other hand has the potential to introduce a continuous disturbance that requires adequate compensation for angle \( \theta_w \). While an estimate of wind would not be assumed available for control, the effects of wind can be anticipated and control terms introduced to provide compensation. A major influence of wind will see the rate of approach altered as the feature transitions towards the image centre. Ideally the rate of approach would be constant up until a point close to the feature where it will gradually reduces to zero ensuring the feature is
centred without overshoot. In order to compensate for the effects of wind on this desired behaviour, a term $V_a$ is introduced to regulate approach velocity, shifting the desired line angle curve of Figure 3.5 vertically to either increase or decrease the rate of approach accordingly,

$$V_a = k_{v1} \left[ \left( 1 + e^{\frac{|T_e| - k_{v2}}{k_{v3}}} \right)^{-1} - 1 \right] \text{sgn}(T_e) - \frac{dT_e}{dt}$$  

(3.8)

While $V_a$ may appear to introduce a number of control terms that would complicate tuning of the controller, each has a simple physical interpretation that should ensure that selection of each is straightforward, as illustrated in Figure 3.7. It is important to note that the derivative of Sensor Track Error would be sensitive to noise and would require filtering prior to use in feedback.

Figure 3.7: Desired Approach Velocity ($V_a$) as a function of Sensor Track Error ($T_e$). Term $k_{v1}$ determines maximum approach velocity, $k_{v2}$ the point of transition and $k_{v3}$ the rate of transition (not illustrated).

Another influence of wind on the tracking process that can be expected is the introduction of steady state tracking error given $\Theta_d(0) \neq 0^\circ$ as a result of $\theta_w$. This is again compensated by shifting the curve of Figure 3.5 vertically, this time with the inclusion of an integral term,

$$R_{T_e} = \int T_e \, dt$$  

(3.9)

The final controller can then be expressed as,

$$\Theta_d(T_e) = \Delta \chi_{\text{max}} \left( 2 \left( 1 + e^{\frac{T_e}{k_r}} \right)^{-1} - 1 \right) + k_r R_{T_e} + k_v V_a$$  

(3.10)
Manoeuvring the aircraft such that $\Theta_{\text{obs}}$ equals $\Theta_d$ can then be achieved through proportional control of rudder minimising error $\Theta_e = \Theta_d - \Theta_{\text{obs}}$, with,

$$\delta_r = k_p \Theta_e$$  \hspace{1cm} (3.11)

Under this design, ailerons, elevators and throttle are free to operate independently of the IBVS controller and thus used to maintain altitude, airspeed and wings level flight. In this way, a conventional autopilot can navigate the UAV to the inspection site, at which time the IBVS controller can take control of rudder and begin tracking. During this time the autopilot would then maintain altitude, airspeed and wings level flight enabling the aircraft to perform STT manoeuvres. The design has the advantage of allowing the two systems to operate independently of one another preserving the reliability of a pre-existing autopilot system that could otherwise be compromised by switching full control to a vision based control system. A general overview of the system is provided in Figure 3.8.

Figure 3.8: Overview of the proposed STT IBVS control system. The design works in conjunction with an independent autopilot system to execute STT manoeuvres utilising visual cues from a downward facing imaging sensor.
3.1 Problem Formulation

3.1.4 Parameter Tuning

Although the controller has a total of eight parameters to tune, a series of steps can be taken to isolate them into groups for a systematic approach to tuning. Fundamental to the controller are three parameters that can provide a basic level of control with others set to zero. The first two, maximum approach angle, $\Delta \chi_{\text{max}}$, and transition rate, $k_s$, from Equation 3.10, are responsible for determining the Desired Line Angle, while proportional term $k_p$ from Equation 3.11 determines the level of commanded rudder that is used to servo Observed Line Angle towards the Desired Line Angle.

With other parameters and wind set to zero, $\Delta \chi_{\text{max}}$ and $k_s$ are tuned to regulate the speed at which the feature moves towards the image centre, where $\Delta \chi_{\text{max}}$ determines the response far from the image centre and $k_s$ the transition towards the centre. Proportional term $k_p$ is tuned at the same time to ensure sufficient rudder is applied to servo the Observed Line Angle towards Desired Line Angle. The next step introduces wind to allow tuning for Approach Velocity, $V_a$, and Residual Track Error, $R_{Te}$.

Beginning with Approach Velocity, term $k_{v_1}$ is set to reflect the approach velocity far from the image centre tuned for the response in no wind, while $k_{v_2}$ and $k_{v_3}$ are set to control the transition as the feature slows towards the image centre. With these terms set, the amount of compensation for approach velocity can then be tuned with parameter $k_v$. Finally parameter $k_r$ is tuned to reduce any steady state error that is observed during steady state tracking.

Following this procedure, controller gains were set to $\Delta \chi_{\text{max}} = 15$, $k_s = 30$, $k_r = -0.035$, $k_v = 0.3$ for (3.10), $k_p = 0.01$ for (3.11) and $k_{v_1} = -40$, $k_{v_2} = 100$, $k_{v_3} = 10$ for (3.8).

The final implemented controller is then given by,

$$
\delta_{rc}(T_e, \Theta_{\text{obs}}) = 0.01 \left[ 30 \left( 1 + e^{\frac{T_e}{30}} \right)^{-1} - 0.035 \int T_e \, dt - \Theta_{\text{obs}} - 15 \right] - 0.3 \left[ 40 \left( 1 + e^{\frac{T_{\text{obs}}-100}{100}} \right)^{-1} \right] \text{sgn}(T_e) + \frac{dT_e}{dt} \right] \tag{3.12}
$$

It should be noted that throughout the tuning process the overall objective was to minimise Sensor Track Error and recentre the feature as fast as possible without incurring overshoot.
3.2 Simulation Environment

To test the performance of the proposed controller, a simulation environment was developed in MATLAB Simulink\textsuperscript{®} that would not only allow flight of a UAV to be simulated, but also allow the generation of images as would be captured by a downward facing, body-fixed imaging sensor. Initially simulations would consider straight sections of infrastructure and the effectiveness of the controller to recentre the feature, while the transition between segments at acute angles would be addressed later in Chapter 6.

In order to generate synthetic imagery, the location of segment end points, as expressed in latitude, longitude and altitude (LLA) ($\phi$, $\lambda$, $h$), are transformed to image plane coordinates ($u$, $v$) through a series of standard photogrammetric transforms [212,213]. These include LLA to Earth Centred Earth Fixed (ECEF), ECEF to Local Vertical, Local Vertical to Body-Fixed, Body-Fixed to Camera Frame and finally Camera Frame to Image Plane. The reader is referred to Appendix B.2.1 for a detailed description of these transformations.

A basic model of power line infrastructure is used to set real world parameters forming 100 m linear segments supported 10 m above the ground by power poles. The catenary of the power line that forms as a result of supporting its own weight is assumed negligible given the overhead vantage from which the feature would be viewed. This allows the power line to be approximated as a linear feature, with its position defined by the location of adjoining poles. Lens distortion is also assumed negligible such that straight line features are likewise projected as straight line features in the 2D image plane.

The projected view of the feature as captured by the onboard sensor is then created by joining the image coordinates of the transformed segment end points. This then allows an estimate of feature parameters $T_e$ and $\Theta_{obs}$ to be made and provided as closed loop feedback for the visual controller. Aircraft response is then simulated through the numerical solution of the nonlinear 6 degree-of-freedom equations of motion using a dynamic model of the Aerosonde\textsuperscript{®} UAV that is included in the AeroSim Blockset by Unmanned Dynamics for MATLAB Simulink [214]. This in turn provides the state variables required for LLA to image plane coordinate transformations.

Autopilot control is emulated through three separate PID loops, regulating airspeed via throttle, altitude via elevator and heading via ailerons. This allows
the UAV to navigate to the inspection area and subsequently provides control required to maintain altitude, airspeed and wings level flight during vision based tracking. Practical limits are placed on control surface deflections to reflect those encountered in reality, with aileron, elevator and rudder deflections restricted between $-45^\circ \leq \delta_a \leq 45^\circ$ and $-30^\circ \leq \delta_e, \delta_r \leq 30^\circ$ respectively. A detailed description of the simulation environment and autopilot subsystem are provided in Appendix B.

3.2.1 BTT PBVS Controller

In order to assess the performance of the proposed STT IBVS controller, a comparison is sought between the closely related vision based technique of Position-Based Visual Servoing (PBVS) that has been presented for similar tracking tasks utilising downward facing imaging sensors, as discussed in Chapter 2.1.1. Although approaches vary, the fundamental idea utilises visual information to provide relative position between UAV and feature that allows the development of a controller in the 3D task space. In terms of data collection, a common factor among proposed solutions is the use of BTT manoeuvres for which the overall impact on sensor FOV and subsequent data collection can be expected to be similar. The PBVS controller implemented in this instance is based on the ‘Good Helmsman’ guidance law developed by Rysdyk [83], commanding desired heading as a function of cross track error, as measured between the aircraft and feature centreline, to command desired heading as follows,

$$\chi_d = \chi_f + \frac{\pi}{2} \left(1 - 2 \left(1 + e^{\frac{C_e}{k_s}}\right)^{-1}\right)$$

(3.13)

where $\chi_d$ is desired bearing, $\chi_f$ bearing of the feature, $C_e$ cross track error and $k_s$ a tuning parameter that determines the rate of approach. This value was tuned to achieve a response that would minimise cross track error as fast as possible without incurring overshoot, a value that determined through trial and error to be $k_s = 25$. Controlling the aircraft on a desired bearing is then achieved through a PID control loop generating desired bank to initiate BTT manoeuvres. For further details regarding these lower level control loops the reader is referred to Appendix B.1.2.
3.3 Test Cases

A series of tests were developed to compare the performance of the proposed STT IBVS controller, under typical operating conditions, to that of related approaches that utilise BTT PBVS control designs. At the beginning of the simulation the UAV would be initialised on a northern heading, 15 m due east of power lines orientated north-south. This provides the onboard sensor with an initial view of the feature as shown in Figure 3.9a, from which the vision based controller would attempt to recentre the feature as shown in Figure 3.9b.

Flight parameters for each scenario, including desired altitude and airspeed, would remain constant for the duration of each test, while gains for each of the autopilot control loops would remain constant for all scenarios.

![Initial view of feature at the beginning of the simulation.](image)

![Desired view of feature during steady state tracking, centred and vertical.](image)

**Figure 3.9:** Comparison of initial and desired view of features during simulation.

The selection of altitude at which an inspection task would take place is highly dependent on the angular FOV of onboard sensors, as these factors together will determine the resulting view of features and the surrounding area. In general, angular FOV can be expected to impose lower limits on altitude, ensuring full width of the inspection region is captured, while minimum spatial resolution requirements necessary to detect the feature will likewise determine upper limits.

For the purpose of demonstration, an ideal pin hole camera model is selected with a horizontal angular FOV of 50° and a sensor resolution of $2048 \times 1536$ pixels, or 3 MP. This limits the lower operating altitude, on account of capturing a 20 m
easement surrounding infrastructure, to 43 m\textsuperscript{i}, while restricting upper altitude to 110 m\textsuperscript{ii}, reflecting a spatial resolution limit of 0.05 m required to detect power lines \[199,215\].

Variations in height can also be expected during operation on account of unmodelled terrain where the autopilot will generally measure and maintain altitude with respect to Mean Sea Level (MSL) that will see height above ground level vary unless updated with a Digital Terrain Model (DTM) or active ranging sensor (e.g. laser or ultrasonic). For this reason operating altitudes would likely include a margin within the limits to cover uncertainty.

Airspeed selection is likely to favour conditions that increase range of the platform, although in some instances may be lowered in an effort to reduce motion and increase data coverage, e.g. image overlap and LiDAR point density \[25\]. With respect to the Aerosonde platform, this range is reflected between airspeeds of 70 km/h (38 kn) and 100 km/h (54 kn). A final test condition would assess the controllers tracking performance in the presence of a constant wind disturbance.

Whilst initial tests would be performed under no wind conditions, subsequent scenarios would introduce the worst case condition of a direct cross wind acting with respect to the feature, selected in this instance at 28 km/h (15 kn). Test cases would then assess the performance of the controller over the full range of expected operating conditions, including combinations of varying altitude, airspeed and wind direction resulting in a total of 5 test cases and 12 scenarios as summarised in Table 3.1.

<table>
<thead>
<tr>
<th>Test Cases</th>
<th>Altitude (m)</th>
<th>Airspeed (km/h)</th>
<th>Windspeed (km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal</td>
<td>50</td>
<td>70</td>
<td>0</td>
</tr>
<tr>
<td>Crosswind</td>
<td>50</td>
<td>70</td>
<td>±28</td>
</tr>
<tr>
<td>Increased Airspeed</td>
<td>50</td>
<td>100</td>
<td>0, ±28</td>
</tr>
<tr>
<td>Increased Altitude</td>
<td>100</td>
<td>70</td>
<td>0, ±28</td>
</tr>
<tr>
<td>Both Increased</td>
<td>100</td>
<td>100</td>
<td>0, ±28</td>
</tr>
</tbody>
</table>

Table 3.1: Test cases developed to assess the performance of the proposed STT IBVS controller under typical operating conditions.

\textsuperscript{i}Assuming a downward facing sensor, Width of Coverage = 2 × Altitude × \tan(FOV/2), where FOV is the horizontal angular FOV of the sensor.

\textsuperscript{ii}Given a 2048 \times 1536 pixel sensor, a spatial resolution of 0.05 m corresponds to a width of coverage of 102.4 m, thus with a horizontal FOV of 50°, altitude is limited to < 109.8 m.
3.4 Results

Results of the first test case are shown in Figure 3.10, with both controllers operating under ideal conditions. Figure 3.10a shows the time history of Cross Track Error, measured as the perpendicular distance from the vehicle to the feature centreline, which is a popular metric used in assessing the performance of lateral tracking controllers. Both controllers are seen to follow similar trajectories as they achieve their respective goals, where the BTT PBVS controller directly minimises cross track error, while the STT IBVS controller indirectly minimises cross track error as a result of recentring the feature in the image plane. Based on this result there would appear to be little difference between the two controllers, however what is not evident is the underlying motion of onboard sensors and the subsequent impact this would have on data collection.

![Graph of Cross Track Error](image1)

(a) Cross Track Error with respect to feature centreline.

![Graph of Sensor Track Error](image2)

(b) Sensor Track Error as measured in the image plane.

**Figure 3.10**: Performance of the proposed STT IBVS controller versus BTT PBVS under ideal operating conditions (No Wind, 70 km/h Airspeed, 50 m Altitude). Also shown are the effects of limiting BTT manoeuvres to ensure features remain visible.

Figure 3.10b shows the time history of Sensor Track Error for both controllers, providing insight into feature motion during UAV flight. A difference between the controllers is now immediately evident, where the BTT PBVS controller is seen to introduce considerable motion within captured imagery compared to that of the STT IBVS controller. This highlights two issues over the use of PBVS control in this context that both originate from the technique deriving control error in the 3D task space.
3.4 Results

First and foremost is the inability to ensure features will remain visible during tracking, as the controller can command manoeuvres that see features leave the sensor FOV, as highlighted during the initial 5 s where the feature leaves the FOV twice as seen by the FOV limits indicated in Figure 3.10b. This has obvious implications for both the tracking and data collection processes that rely on a continuous view of features from onboard sensors. Restricting manoeuvres to avoid these situations is a solution that has been previously proposed for similar control tasks [16, 119], and is included for comparison in the results of Figure 3.10b (Bank-Limited) with variable limits for bank angle, $\phi$, calculated as,

$$-\frac{\alpha}{2} + \arctan\left(\frac{|C_e|}{h}\right) < \phi < \frac{\alpha}{2} + \arctan\left(\frac{|C_e|}{h}\right)$$  (3.14)

where $\alpha$ is horizontal angular FOV, $C_e$ cross track error and $h$ altitude.

The impact of introducing these limits is seen to slow the controllers response, and while the feature remains visible, the view is not necessarily ideal, with the feature seen on the very edge of the image frame over the initial 5 s of the simulation. The second issue concerning PBVS control for data collection, although not as obvious from Figure 3.10b, is the rate of feature motion and the lack of control PBVS offers to reduce this. The BTT manoeuvre in particular is seen to induce rapid motion of the feature through the image plane that can reduce the quality of data (e.g. motion blur in captured images), affecting both visual tracking and data collection processes.

Comparing this to the proposed STT IBVS controller, the response in Sensor Track Error is seen to be far more desirable, with the controller recentring the feature in one smooth motion. Assessing feature motion in a quantitative manner is proposed through the summation of the derivative of Sensor Track Error, representing motion of features between successive frames. Using this metric, the STT IBVS controller is found to induce 86% less motion to that of the BTT PBVS controller, and 57% to that of the Bank-Limited controller. Another important metric to consider is the time to recentre. While settling time would generally provide a metric for comparison, it can prove misleading when assessing the performance in the context of data collection.

This is highlighted between a comparison of Bank-Limited PBVS and STT IBVS that both achieve 5% settling times of 10 s, however display distinct differences in response. For this reason an additional metric is introduced in the form
of Total Error, measured as the area under each curve allowing a quantitative assessment of Sensor Track Error over time. Using this metric reveals the difference in response, where the STT IBVS controller is found to reduce Total Error by a factor of 40%. It should be noted that the two metrics can be expected to share an inverse relationship, with total error reducing on account of a faster response that will increase feature motion.

A point to note regarding the simulation of the proposed controller is ensuring angles of sideslip stay within suitable bounds for the aircraft model. As discussed in Appendix B.1.1, the stability and control coefficients of the chosen model are constant and therefore can only be expected to provide accurate simulation over a linear region. In the case of sideslip these bounds are considered to be within ±15°. Figure 3.11a shows the angle of sideslip induced by the STT IBVS controller, where it can be seen to remain well within these bounds, initially reaching a peak of 12° before falling within ±7°.

Another point to note is the effect of introducing angle of sideslip in the calculation of Desired Line Angle that during the development of the controller was neglected from Equation (3.7). This was on account of not being able to observe the angle from imagery and removed under the assumption that the angle would remain small. Figure 3.11b shows the result of including sideslip in

![Figure 3.11](attachment:image.png)

(a) Angle of sideslip induced during STT IBVS control response. (b) Comparison of STT IBVS with addition of sideslip angle in control law.

**Figure 3.11:** Angle of sideslip induced by the STT IBVS controller, that is seen to remain within the ±15° limits, and the effect of including the angle in the calculation of Desired Line Angle that in practice would not be available.
the calculation of Desired Line Angle, where the angle is directly added to the control law of (3.10) in the form of,

$$\Theta_d(T_e) = \Theta_d(T_e)|_{old} + \beta$$ (3.15)

where the introduction of the angle is seen to provide marginal improvement in the final response, avoiding overshoot that occurs as the feature reaches the image centre at 8 s. This coincides with sideslip induced by the controller to slow the final approach and explains the original controller that excludes the angle undercompensating and incurring overshoot.

A similar result could be expected at the beginning of the scenario where a larger angle of sideslip is induced, however as a result of both controllers initially commanding control in excess of rudder limits, the response during this time is very similar. It can therefore be seen that the exclusion of the angle from the calculation of Desired Line Angle will have minimal impact on the controller.

The next series of simulations would test the proposed controller under realistic operating conditions, in particular, the ability to reject an external disturbance introduced by a constant wind. The conditions under which the controller would be tested would reflect the worse case scenario of tracking in the presence of a direct crosswind with respect to the feature. Wind speed was chosen to reflect a moderate breeze of 28 km/h (15 kn) acting from the two extremes, west-to-east and east-to-west. Both controllers are required to compensate for wind indirectly as neither is provided with, nor assumes, knowledge of the wind.

In terms of the PBVS controller, the influence of wind is expected to introduce steady state Cross Track Error and is therefore compensated with the addition of integral control, while the design of the STT IBVS controller includes terms $V_a$ and $R_T_e$ to manage the effects of wind. The effect of the two wind scenarios can be expected to be very different. In one case the UAV will fly into the wind, requiring increased control effort to recentre the feature, while the second scenario will see the UAV drift towards the feature as it flies with the wind, requiring reduced control effort to avoid possible overshoot.

Figures 3.12 (a - b) and (c - d) show the response of the aircraft flying into west-to-east and east-to-west winds respectively, as indicated by arrows on each graph. From the perspective of Cross Track Error, the BTT PBVS controller would appear to handle both wind conditions very well, following a similar trajectory
to that of the ideal case in Figure 3.10. *Cross Track Error* is once again however found to conceal many of the underlying data collection issues that now includes the feature appearing offset in the image plane during steady state tracking. This results from steady state pitch required to maintain lift that sees the body-fixed sensor angled slightly forward and ahead of the aircraft.

![Cross Track Error](image1)

![Sensor Track Error](image2)

(a) Cross Track Error response as a result of West-to-East wind.

(b) Sensor Track Error response as a result of West-to-East wind.

(c) Cross Track Error response as a result of East-to-West wind.

(d) Sensor Track Error response as a result of East-to-West wind.

**Figure 3.12**: Performance of the proposed IBVS controller versus PBVS controller in the presence of a 28 km/h (15 kn) cross wind with respect to feature (as indicated by arrows). Airspeed and altitude remain unchanged at 70 km/h and 50 m respectively.

Under ideal conditions this does not present an issue as *Aircraft Heading* reflects *Course Over Ground*, thus the sensor is simply angled further along the feature. However when flown in the presence of wind, *Aircraft Heading* no longer
aligns with *Course Over Ground* and as a result, the aircraft flies at an angle to the feature. This leads to downward facing sensors being pointed slightly away from the feature when the aircraft is flown directly over the feature.

The STT IBVS controller is seen to compensate for this by flying at a steady state *Cross Track Error* of approximately 2 m as seen in Figures 3.12 (a-c). Overall, the controller is seen to effectively handle both wind scenarios, with settling times increased on account of integral control used to compensate of wind. The controller is seen to encounter more issues flying with the wind, as in the case of Figure 3.12d, where the controller overshoots, to that of flying into the wind, as in the case of Figure 3.12b, where the effect is a small increase in settling time.

Although the STT IBVS controller incorporates measures to compensate for approach velocity, and thus limit overshoot, the main effects of wind are addressed through integral control that only takes affect after the controller attempts to recentre the feature. When flying into the wind the effect is immediate as wind slows the approach of the UAV causing integral control to begin early. However, when flying with the wind, an attempt to recentre the feature results in overshoot that delays integral control taking affect. The introduction of wind is found to increase feature motion of the STT IBVS controller by 10% and 54% respectively for west-to-east and east-to-west cases, although still provides improvement over BTT PBVS, reducing feature motion by 80% and 83% in respective wind test cases and 50% and 40% with bank limits.

The next series of tests would assess robustness of the controller to varying operating conditions, with control gains remaining the same as those previously set. Figure 3.13b shows the results of increasing UAV airspeed from 70 km/h (38 kn) to 100 km/h (54 kn) under scenarios of no wind and direct cross winds of 28 km/h (15 kn) acting west-to-east and east-to-west. The increase in airspeed is actually seen to improve performance of the controller under both ideal and wind conditions, reducing *total error* by factors of 32%, 39% and 46% for no wind, west-to-east and east-to-west winds respectively.

The improved performance can be attributed to the increase in turn rate that is achieved at higher airspeeds for a given angle of sideslip. Increased airspeed also has the benefit of reducing the angle between aircraft heading and course over ground, that sees the observed line angle during steady state tracking reduced from 23° to 16° in the presence of a 28 km/h cross wind. This leads to
reduced overshoot and shorter settling times given less compensation is required by integral control. Figure 3.13c shows the results of increasing altitude, and thus the distance between the sensor and feature, from 50 m to 100 m.

![Chart](image)

(a) Original control response of STT IBVS (50 m Altitude, 70 km/h Airspeed)

(b) Control response at increased Airspeed (50 m Altitude, 100 km/h Airspeed)

(c) Control response at increased Altitude (100 m Altitude, 70 km/h Airspeed)

(d) Both Airspeed and Altitude increased (100 m Altitude, 100 km/h Airspeed)

**Figure 3.13:** Performance of STT IBVS controller under varying flight conditions compared to (a) original response, (b) increased airspeed 70 → 100 km/h (c) increased altitude 50 → 100 m (*note change in time scale*), (d) increases to both airspeed and altitude. Each scenario was repeated under no wind and direct crosswinds (±28 km/h).

From the perspective of Sensor FOV, variations in height effectively see the relationship between Sensor Track Error and Cross Track Error scaled, while the relationship between Course Over Ground and Observed Line Angle remain unchanged. This is immediately evident from the results, where Sensor Track Error
3.4 Results

is seen to decrease from approximately 400 pixels to 200 pixels, while the simulation is still initialised with a Cross Track Error of 15 m. The UAV subsequently has the same distance to move towards the feature, while only observing half the error in the image plane, effectively detuning the controller and slowing the overall response. This is reflected in 5% settling times that are seen to increase by factors of 127%, 26% and 144% for no wind, west-to-east and east-to-west wind respectively, where it should be noted that the time scale of Figure 3.13c is scaled from 20 s to 60 s to accommodate the responses.

Overshoot is also seen to increase, particularly in the case of east-to-west winds where percent overshoot is observed at 88%. This raises concern of overshoot increasing at larger values of Sensor Track Error, hence an additional scenario is included initialising the UAV at twice the Cross Track Error (2CTe), resulting in Sensor Track Error approximately equal to the original scenario. Percent overshoot is now seen to decrease, however on a note of interest, reaches the same overall value of approximately 180 pixels. Adding to this, the same value is observed in the original response of Figure 3.13a.

This can be explained on account of each scenario initially assuming no wind that sets an incorrect final value of Desired Line Angle about which the controller drives towards. This does not prevent the controller reaching the necessary line angle, rather the value of Sensor Track Error at which this angle is calculated by Desired Line Angle is non-zero and leads to the consistent overshoot between scenarios. Thus as opposed to ‘overshooting’, the controller is actually driven towards this angle and would stay at this value without integral control, hence the same response can be expected irrelevant of initial conditions for the same wind and airspeed. Coincidentally, the angle at which maximum overshoot occurs is the angle at which the feature will be observed during steady state tracking that could potentially prove useful in the estimation of wind.

While the increase in altitude is seen to have a negative impact on the performance of the STT IBVS controller, it should be noted that in practice variations of altitude on this scale would be rare and if anticipated would more likely see the controller retuned. The final scenario would assess the combined affect of increasing both altitude and airspeed, with the results shown in Figure 3.13d. At this point the controller is far from the original operating conditions about which the controller was tuned, although is still seen to command a desirable response. The reduced performance encountered at higher altitudes is seen to be compensated
by the improved performance that was gained with increased airspeed. Settling times are seen to be considerably reduced compared to increased altitude alone, with total error reduced by 50%, 29% and 60% for no wind, west-to-east and east-to-west winds respectively. Overshoot is once again improved as a result of increased airspeed, that sees the final line angle reduced from $23^\circ$ to $16^\circ$.

In summary, the proposed STT IBVS controller has been shown to improve data collection conditions during tracking, offering a direct means of control over the inspection process. This improves over BTT PBVS techniques that are commonly used among related approaches, reducing both feature motion and total error by allowing the feature to be centred in one smooth motion. The controller also avoids issues of features leaving the sensor FOV during tracking that is an issue with PBVS techniques. Although this can be avoided by imposing limits on manoeuvres, the overall result on data collection is not necessary ideal, as is shown in the case of Bank-Limited BTT manoeuvres where features are observed at the very edge of the image frame during the transition.

Constant wind disturbances are shown to be effectively compensated through terms to control approach velocity and steady state error, although on account of initially assuming no wind, incurs overshoot when the aircraft is repositioning in the same direction as the wind. The controller has also been shown to be robust to variations in both airspeed and altitude. Performance is actually improved on account of increased airspeed and would be recommended where permitted. Increased altitude on the other hand was found to degrade performance, slowing the response of the controller due to relative scaling of Sensor Track Error, that would be better served re-tuning the controller if large variations are expected.

It should be noted that this form of control could lead to steady state sideslip over certain periods of time. This may affect the UAVs onboard Inertial Measurement Unit (IMU), where the system could erect to a false horizon, although should be alleviated given an integrated GPS/INS navigation solution is used.
3.5 Summary

This chapter has presented a novel solution to the simultaneous tracking and data collection problem, providing control of feature position and motion as observed by an inspection sensor. The development builds upon the concept of observing relative alignment between the UAV and infrastructure as a function of feature orientation as viewed from a downward facing imaging sensor. This allows the derivation of a Desired Line Angle based on the view of infrastructure as would be observed as the UAV follows a desired trajectory.

Control of the UAV is then fulfilled by servoing the vehicle to meet the desired view, for which an Image Based Visual Servo control solution is developed, altering the observed orientation of features through wings-level Skid-to-Turn manoeuvres initiated through rudder. The overall result is a controller that can operate independently of an autopilot that would provide navigation enroute to the inspection area and maintain airspeed, altitude and wings-level attitude during visual tracking.

Controller performance has been demonstrated through a series of simulations with comparison to a Position-Based Visual Servo controller utilising Bank-to-Turn manoeuvres that is common in literature. The controller was tested over a range of practical operating conditions, including variations in wind, airspeed and altitude for which the controller was found to consistently outperform the BTT PBVS controller, centring the feature in a timely fashion whilst ensuring features remained within the sensor FOV. Aside from ensuring features remain visible during tracking, the proposed STT IBVS controller was also shown to provide a solution for reducing feature motion that can potentially degrade data quality that is not addressed in other published work.
Forward-Slip IBVS

The following chapter presents a novel solution to the simultaneous tracking and data collection problem utilising the full dynamics of the aircraft to provide increased performance at reduced control effort. In the previous chapter, a novel solution was presented through IBVS that required minimal interaction between a pre-existing autopilot, offering visual control through rudder alone. Whilst effective, suppression of unwanted motion is achieved indirectly, restricting the aircraft to wings-level skid-to-turn manoeuvres that limit the overall performance of the controller.

The following chapter seeks to provide a solution given the controller has full access to the autopilot. While this requires full integration between the controller and autopilot that the earlier solution avoided, the solution does allow the controller to fully utilise the dynamics of the aircraft to the advantage of data collection. Formulation of the control design begins with the development of an interaction matrix to model motion of image features as a function of camera motion. This is then augmented with the aircraft dynamics to allow the development of a Linear-Quadratic Regulator (LQR) that allows control of both the inspection sensor FOV and the reduction of unwanted motion.
The optimal solution that arises from the LQR design is found to utilise a *Forward-Slip* manoeuvre in place of *Skid-to-Turn*, where the new manoeuvre allows the UAV to maintain a constant angle of bank while altering course over ground. This allows the controller to angle body-fixed sensors towards the feature while simultaneously altering aircraft heading to move towards the feature centreline. Performance of the controller is assessed through simulation, accounting for variations in both airspeed and altitude.

### 4.1 Problem Formulation

The following section details the development of a state space representation for the visual servo system that includes both modelling aircraft dynamics and the relationship between aircraft motion and subsequent view of infrastructure. The primary contribution of the work surrounds the augmentation of the aircraft dynamic model with the interaction matrix of the image features that allow a controller to be developed that achieves platform stabilisation and feature tracking simultaneously.

#### 4.1.1 Feature Representation

At the foundation of the IBVS control design is appropriate selection of features and their respective representation in both the 3D camera frame and 2D image plane. Infrastructure is once again assumed to be extracted and modelled as a single line feature as was defined in Chapter 3 and illustrated in Figure 3.3. Although the representation of linear features in both 2D and 3D frames are numerous, careful selection can offer convenient approximations and assumptions that reduce the overall complexity of the derivation, while others can provide practical measures that allow the formation of logical control objectives.

A popular form of 3D line expression used in IBVS control development is based on the intersection of planes, originating from the work of Espiau et al. [165], expressed as,

\[
L = \begin{cases} 
  a_1X + b_1Y + c_1Z + d_1 = 0 \\
  a_2X + b_2Y + c_2Z + d_2 = 0
\end{cases}
\]  

(4.1)
The representation has been popular in IBVS solutions for UAVs, as the development of an interaction matrix is satisfied with knowledge of only one of the two planes [178,187,189]. This is particularly useful for a UAV viewing ground based objects over relatively flat terrain where one plane can be approximated as parallel to the aircraft body axes $o_xb$-$oyb$ (Figure A.1), for which a transformation between camera and body-fixed coordinates is generally available. In the context of inspection, many features will either be ground based (pipelines, roads) or supported at a fixed distance from the ground (power lines) for which these benefits can be utilised and hence the adoption of the model for the following developed.

![Diagram of line features and time histories](image)

(a) Line Feature at $t = t_1$ expressed in normal form by $(\rho_1, \theta_1)$
(b) Line Feature at $t_2 (t_1 + \Delta t)$ having moved past image centre.
(c) Time history of $\rho$ for $t_1 < t < t_2$
(d) Time history of $\theta$ for $t_1 < t < t_2$

**Figure 4.1:** Discontinuity of normal form parameters $\rho$ and $\theta$ as a feature passes image centre that is not ideal when considering the terms as control variables.

In terms of 2D expressions, line equations can take on any number of forms, including **Slope-Intercept**, $y = mx + c$; **Point-Slope**, $y - y_1 = m(x - x_1)$; **Normal Form**, $\rho = x \cos \theta + y \sin \theta$; or **General Form**, $Ax + By + C = 0$. From the
perspective of control, the *normal form* offers a convenient geometric relationship that separates the representation into two logical control objectives, with \( \rho \) providing a measure of the features displacement from the image centre and the angle \( \theta \) providing the direction of the line within the image frame. One issue with the use of \( \rho \) and \( \theta \) in expressing the extracted line feature however arises when the feature passes the image plane centre, as is likely to occur regularly during tracking. Figure 4.1 highlights the issue, where both \( \rho \) and \( \theta \) are shown over the time period \( t_1 < t < t_2 \) during which a line feature passes the image centre.

It can be seen that both \( \rho \) and \( \theta \) encounter discontinuities as the feature passes the image centre, which is undesirable given it occurs about the desired feature position and the parameters are intended as control variables. An alternate line representation is sought to overcome this issue, and is derived through a similar line representation as was developed in Section 3.1.3 with *Sensor Track Error*, \( T_e \), that provides a similar measure of perpendicular distance from image centre as \( \rho \) although adopts sign relative to feature position left \((-T_e)\) and right \((+T_e)\) of image centre, and *Line Angle*, \( \Theta_l \), that provides a relative measure of line orientation independent of feature position measured clockwise from image vertical, as shown in Figure 4.2.

![Diagram](image.png)

**Figure 4.2:** Use of \( \pm T_e, \Theta_l \) to avoid issues of *normal form* \((\rho, \theta)\). Line orientation is seen to remain constant as the feature moves from position (1) to (2), while \( \theta \) varies \((e.g. \ \theta_1 \neq \theta_2)\), while \( T_e \) is continuous as the feature passes the image centre.
Mathematically, the alternate line representation using $T_e$ and $\Theta_l$ can be expressed as,

$$T_e = -x \sin \Theta_l + y \cos \Theta_l$$  \hspace{1cm} (4.2)

It should be noted that this moves the discontinuity as opposed to eliminating it, with feature parameters $T_e$ and $\Theta_l$ experiencing a discontinuity as the feature rotates past horizontal. In the context of tracking however this should not be an issue as the aircraft would not approach at such an angle. Adopting a relative orientation metric is also seen to provide a more intuitive control metric, with $\Theta_l = 0^\circ$ reflecting a feature passing vertically through the image frame, remaining constant as the aircraft moves over the line.

### 4.1.2 Interaction Matrix

Having defined the line representation in both 3D and 2D frames, it is now possible to derive the interaction matrix, or image Jacobian, $L_s \in \mathbb{R}^{k \times m}$, relating feature motion as a result of camera motion through,

$$\dot{s} = L_s T_c$$  \hspace{1cm} (4.3)

where $s$ represents a vector of $k$ image feature parameters and $\dot{s}$ the corresponding image feature parameter velocities, while $T_c$ represents camera motion through a velocity screw that defines both translational and rotational velocities for the camera’s $m$ degrees-of-freedom. For a camera with 6 degrees-of-freedom, $T_c$ can be expressed as,

$$T_c = [U_c \ V_c \ W_c \ P_c \ Q_c \ R_c]^T$$  \hspace{1cm} (4.4)

where $(U_c, P_c)$, $(V_c, Q_c)$, and $(W_c, R_c)$ are the respective translational and rotational velocities for camera axes $o_xc$, $o_y c$ and $o_z c$ as defined in Figure A.1.

Derivation of the interaction matrix follows a similar approach as presented by Espiau et al. [165], given the same 3D line representation is adopted, although differs given the alternate 2D representation of Eqn. (4.2) that expresses line pose in terms of $T_e$ and $\Theta_l$ as,

$$T_e = -x \sin \Theta_l + y \cos \Theta_l$$  \hspace{1cm} (4.5)

The derivation begins by taking the derivative of the line equation with respect
to time that results in,

\[
\begin{bmatrix}
1 & x \cos \Theta_l + y \sin \Theta_l
\end{bmatrix}
\begin{bmatrix}
\dot{T}_e \\
\dot{\Theta}_l
\end{bmatrix} = 
\begin{bmatrix}
- \sin \Theta_l & \cos \Theta_l
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix}
\]

(4.6)

Recognising \((x, y)\) as points on the line, motion of those points can be expressed through the interaction matrix of a point feature. A derivation is included in Section A.4, where the motion of a point feature, \((p_x, p_y)\), as projected from its physical location expressed in camera coordinates, \((P_x, P_y, P_z)\), can be expressed as a result of camera motion through,

\[
\begin{bmatrix}
\dot{p}_x \\
\dot{p}_y
\end{bmatrix} =
\begin{bmatrix}
-f/P_z & 0 & p_x/P_z & p_xp_y/f & -f - p_x^2/f & p_y \\
0 & -f/P_z & P_y/P_z & f + p_y^2/f & -p_xp_y/f & -p_x
\end{bmatrix}
\begin{bmatrix}
T_e \\
Z
\end{bmatrix}
\]

(4.7)

Substituting Equation (4.7) into Equation (4.6) then allows line motion to be expressed as a function of the camera velocity screw, \(T_c\), that then allows the for the expression,

\[
\begin{bmatrix}
1 & x \cos \Theta_l + y \sin \Theta_l
\end{bmatrix}
\begin{bmatrix}
\dot{T}_e \\
\dot{\Theta}_l
\end{bmatrix} = 
\begin{bmatrix}
\frac{f \sin \Theta_l}{Z} \\
\frac{-f \cos \Theta_l}{Z} \\
\frac{T_c}{Z} \\
\frac{T_e y + f^2 \cos \Theta_l}{f} \\
\frac{-T_e x - f^2 \sin \Theta_l}{f} \\
\frac{-(x \cos \Theta_l + y \sin \Theta_l)}{f}
\end{bmatrix}
\]

(4.8)

Rearranging Equation (4.1) and assuming an ideal camera model, as detailed in Section A.3, with the principal point, \((c_x, c_y)\), at the image centre and uniform pixels such that \(\eta = 1\) and \(\tau = 0\), allows terms relating to feature depth, \(Z\), to be replaced with,

\[
f/Z = -(a_i x + b_i y + f c_i)/d_i
\]

where line parameters can be taken from either plane, i.e. \(i = \{1, 2\}\).
Substituting into Equation (4.8) then results in,

\[
\begin{bmatrix}
1 & x \cos \Theta_l + y \sin \Theta_l
\end{bmatrix}
\begin{bmatrix}
T_e \\
\dot{\Theta}_l
\end{bmatrix}
= 
\begin{bmatrix}
\frac{(a_i x + b_i y + c_i f) \sin \Theta_l}{d_i} \\
\frac{(a_i x + b_i y + c_i f) \cos \Theta_l}{d_i} \\
- \frac{T_e (a_i x + b_i y + c_i f)}{f d_i} \\
\frac{T_e y + f^2 \cos \Theta_l}{f} \\
- \frac{T_e x - f^2 \sin \Theta_l}{f} \\
-(x \cos \Theta_l + y \sin \Theta_l)
\end{bmatrix}
\]

Terms relating to \( y \) can also be removed recalling the 2D line representation of Equation (4.2), that can be rearranged such that,

\[
x = \frac{y \cos \Theta_l - T_e}{\sin \Theta_l}
\]

thus leading to,

\[
\begin{bmatrix}
1 & \frac{1}{\sin \Theta_l} y - \frac{T_e}{\tan \Theta_l}
\end{bmatrix}
\begin{bmatrix}
\dot{T}_e \\
\dot{\Theta}_l
\end{bmatrix}
= 
\begin{bmatrix}
-K_c y + K_4 \\
\frac{K_c}{\tan \Theta_l} y - \frac{K_4}{\tan \Theta_l} \\
- \frac{K_c T_e}{f \sin \Theta_l} y + \frac{K_4 T_e}{f \sin \Theta_l} \\
\frac{T_e}{f} y + f \cos \Theta_l \\
- \frac{T_e}{f \tan \Theta_l} y + \frac{T_e^2 + (f \sin \Theta_l)^2}{f \sin \Theta_l} \\
- \frac{1}{\sin \Theta_l} y + \frac{T_e}{\tan \Theta_l}
\end{bmatrix}
\]

where,

\[
K_c = \frac{a_i \cos \Theta_l + b_i \sin \Theta_l}{d_i} \quad K_4 = \frac{a_i T_e - c_i f \sin \Theta_l}{d_i}
\]
Removing dependence on $y$ is now possible recognising that each side of Equation (4.9) can be rearranged in the form $\alpha y + \beta$,

\[
\frac{\dot{\Theta}_l}{\sin \Theta_l} y + \frac{T_e}{\tan \Theta_l} \dot{T}_e = \begin{bmatrix} -K_c & K_c \tan \Theta_l & -K_c T_e \sin \Theta_l \tan \Theta_l \\ K_c & -K_c \tan \Theta_l \sin \Theta_l & T_e \sin \Theta_l \tan \Theta_l \\ -K_c T_e \cos \Theta_l & T_e \sin \Theta_l \cos \Theta_l & -1 \end{bmatrix}^T \begin{bmatrix} K_4 \\ -K_4 \tan \Theta_l \\ K_4 T_e \sin \Theta_l \tan \Theta_l \\ \frac{K_4}{f} \sin \Theta_l \tan \Theta_l \\ \frac{T_e}{f} \sin \Theta_l \tan \Theta_l \\ -1 \sin \Theta_l \end{bmatrix}^T T_e y + \begin{bmatrix} K_4 \\ -K_4 \tan \Theta_l \\ K_4 T_e \sin \Theta_l \tan \Theta_l \\ \frac{K_4}{f} \sin \Theta_l \tan \Theta_l \\ \frac{T_e}{f} \sin \Theta_l \tan \Theta_l \\ -1 \end{bmatrix}^T T_e
\]

In order to satisfy $\alpha_1 y + \beta_1 = \alpha_2 y + \beta_2$, $\forall y \in \mathbb{R}$, term $\alpha_1$ must equal $\alpha_2$ and likewise $\beta_1 = \beta_2$. Term $\dot{\Theta}_l$ is therefore seen to equal,

\[
\dot{\Theta}_l = \begin{bmatrix} -K_c \sin \Theta_l & K_c \cos \Theta_l & -K_c T_e \sin \Theta_l \tan \Theta_l \\ \frac{K_4}{f} \sin \Theta_l \tan \Theta_l \\ \frac{T_e}{f} \sin \Theta_l \tan \Theta_l \\ -1 \sin \Theta_l \end{bmatrix}^T T_e
\]

while $\dot{T}_e$ can be expressed as,

\[
\dot{T}_e = \begin{bmatrix} K_4 \\ -K_4 \tan \Theta_l \sin \Theta_l \tan \Theta_l \\ K_4 T_e \sin \Theta_l \tan \Theta_l \\ f \sin \Theta_l \tan \Theta_l \\ f \cos \Theta_l \tan \Theta_l \\ T_e^2 + (f \sin \Theta_l)^2 \tan \Theta_l \\ T_e \sin \Theta_l \tan \Theta_l \\ T_e \cos \Theta_l \tan \Theta_l \\ -1 \end{bmatrix}^T T_e
\]

that simplifies to,

\[
\dot{T}_e = \begin{bmatrix} K_a \sin \Theta_l & -K_a \cos \Theta_l \frac{K_a T_e}{f} & K_b \cos \Theta_l & K_b \sin \Theta_l & 0 \end{bmatrix}^T T_e
\]
where,

\[ K_a = \frac{T_e (a_i \sin \Theta_l - b_i \cos \Theta_l) - c_i f}{d_i} \]

\[ K_b = \frac{f^2 + T_e^2}{f} \]  

(4.11)

Finally the interaction matrix of the line feature can be expressed as,

\[
\begin{bmatrix}
\dot{T} \\
\dot{\Theta}_l
\end{bmatrix} =
\begin{bmatrix}
K_a S \Theta_l & -K_a C \Theta_l & K_a K_d & K_a C \Theta_l & K_b S \Theta_l & 0 \\
-K_c S \Theta_l & K_c C \Theta_l & -K_c K_d & K_c S \Theta_l & -K_d C \Theta_l & -1
\end{bmatrix} T_e
\]  

(4.12)

where \( C \Theta_l \) express \( \cos \Theta_l \) respectively, while terms \( K_{a,b} \) are given by (4.11), \( K_c \) from (4.10), while \( K_d = T_e / f \). Remaining terms \( a_i, b_i, c_i & d_i \) are taken from either plane that define the line feature in the 3D camera frame.

### 4.1.3 Aircraft Model

Having identified a relationship between camera motion and perceived motion of features in the image plane, attention can turn to modelling motion of the camera. Given the problem considers body-fixed sensors this is relatively straight forward as the camera will simply inherit motion of the aircraft assuming the sensor has a rigid mount. Common notation for body-fixed axes are adopted here, with \( o_x b, o_y b \) and \( o_z b \) forming a right hand orthogonal axis system centred at the aircraft centre of gravity (c.g.) with axis \( o_x b \) directed along the fuselage towards the aircraft nose, \( o_y b \) directed along the right wing and \( o_z b \) directed below, as depicted in Figure A.1. Likewise the camera frame is defined with axes \( o_x c, o_y c \) and \( o_z c \) forming a right hand orthogonal axis system centred at the camera centre with \( o_z c \) aligned with the optical axis, and \( o_x c, o_y c \) aligned with the vertical and horizontal axes of the image plane respectively, again depicted in Figure A.1.

For a fixed wing platform it is not uncommon to mount a downward facing sensor close to the aircraft c.g. from which position translation between camera and aircraft frames can be assumed negligible. Alignment between aircraft and camera frames is implied when utilising a ‘downward facing’ sensor, where the term is used to infer sensor orientation achieved during steady state flight. Under a steady straight, symmetric, wings level flight condition this infers alignment of the sensor with the body axes, where LOS is aligned with the \( o_z b \) axis. It should

---

\(^1\)To avoid confusion, the *right* wing can also be considered the *starboard* wing.
be noted that steady state pitch required to maintain an angle of attack will see body axis $oz_b$ angled forward of vertical by an equal amount, as illustrated in Figure 4.3. Compensating for this angle however would prove impractical as the angle varies as a function of airspeed and weight that vary during the course of flight. Thus an approximation for $downward$ facing is made with the sensor orientated with body axis $oz_b$. Under these assumptions the camera velocity screw, $T_c$, can then be considered equal to the state vector of the UAV.

Dynamics of the UAV platform can be described through six nonlinear equations of motion (4.13) and three auxiliary equations (4.14); a derivation that is commonly provided in aerodynamic text [208–210,216], and included in Appendix A.2. The derivation is made under the general assumption of a rigid airframe, symmetrical about the X-Z plane, with constant mass and mass distribution, with the Earth considered an inertial reference frame.

\[
\begin{align*}
X &= m(\dot{U} - RV + QW) \\
Y &= m(\dot{V} - PW + RU) \\
Z &= m(\dot{W} - QU + PV) \\
L &= I_x \ddot{P} - \left(I_y - I_z\right)QR - I_{xz}(PQ + \dot{R}) \\
M &= I_y \dot{Q} + \left(I_x - I_z\right)PR + I_{xz}(P^2 - R^2) \\
N &= I_z \ddot{R} - \left(I_x - I_y\right)PQ + I_{xz}(QR - \dot{P}) \\
P &= \dot{\Phi} - \dot{\Psi} \sin \Theta \\
Q &= \dot{\Theta} \cos \Phi + \dot{\Psi} \cos \Theta \sin \Phi \\
R &= \dot{\Psi} \cos \Theta \cos \Phi - \dot{\Theta} \sin \Phi
\end{align*}
\]
Where,

\[ X, Y, Z \] - Forces acting along Body Axes
\[ L, M, N \] - Moments acting about Body Axes
\[ U, V, W \] - Velocity expressed in Body Axes
\[ P, Q, R \] - Angular Rates about Body Axes
\[ I_x, I_y, I_z \] - Moments of Inertia about Body Axes
\[ I_{xz} \] - Product of Inertia
\[ \Theta, \Phi, \Psi \] - Euler Angles

Forces and moments acting on the airframe can then be considered as the sum of individual contributions arising from aerodynamic effects \((a)\), gravitational force \((g)\), control surface deflections \((c)\), power effects \((p)\) and atmospheric disturbances \((d)\). The resulting expression, using linear acceleration along axis \(o_x\) as an example, is then,

\[
m(\ddot{U} - RV + QW) = X_a + X_g + X_c + X_p + X_d
\]

The equations of motion as expressed in (4.13) are however non-linear and their solution by analytical means is generally not practical, while terms on the left hand side of the equation can be particularly difficult to express in terms of generalised motion \([210]\). A common solution is to linearise the equations of motion, approximating motion as perturbations about mean motion defined by an equilibrium or trimmed flight condition. Forces, moments and motion variables are then expressed as,

\[
U \equiv U_0 + u
\]

where mean motion is denoted with subscript zero \((0)\) and dynamic motion denoted with a lower case letter. For a full set of expressions the reader is referred to Appendix A.2.1 (A.7, A.8), which also details the development of the linearised equations of motion for a general trim condition given by equation (A.11).

During inspection, an ideal trim condition is \(\text{Straight} \ (\dot{\Theta}_0, \dot{\Phi}_0, \dot{\Psi}_0 = 0)\), \(\text{Level} \ (\Phi_0, \gamma_0 = 0)\), \(\text{Symmetric} \ (V_0 = 0)\) flight. This not only provides an efficient flight configuration, but also sees a decoupling between longitudinal motion variables \((u, w, q, \theta)\) and lateral motion variables \((v, p, r, \phi, \psi)\) that is particularly useful in control design. The full development including approximations, assumptions
and formation of the state space representations is presented in Appendix A.2. Longitudinal motion can then be expressed in terms of stability derivatives as,

\[
\begin{bmatrix}
\dot{u} \\
\dot{w} \\
\dot{q} \\
\dot{\theta}
\end{bmatrix}
= \begin{bmatrix}
X_u & X_w & 0 & -g \\
Z_u & Z_w & V_P & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
u \\
w \\
q \\
\theta
\end{bmatrix}
+ \begin{bmatrix}
X_{\delta e} & X_{\delta th} \\
Z_{\delta e} & Z_{\delta th} \\
M_{\delta e}^* & M_{\delta th}^* \\
0 & 0
\end{bmatrix} \begin{bmatrix}
\delta_e \\
\delta_{th}
\end{bmatrix}
\] (4.15)

where,

\[M' = M + M_w Z_e\] \[M_q^* = M_q + M_w V_P\]

While lateral motion is likewise described by,

\[
\begin{bmatrix}
\dot{v} \\
\dot{p} \\
\dot{r} \\
\dot{\phi}
\end{bmatrix}
= \begin{bmatrix}
Y_v & 0 & -V_P & g \\
L'_v & L'_p & L'_r & 0 \\
N'_v & N'_p & N'_r & 0 \\
0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
v \\
p \\
r \\
\phi
\end{bmatrix}
+ \begin{bmatrix}
Y_{\delta a} & Y_{\delta r} \\
L'_{\delta a} & L'_{\delta r} \\
N'_{\delta a} & N'_{\delta r} \\
0 & 0
\end{bmatrix} \begin{bmatrix}
\delta_a \\
\delta_r
\end{bmatrix}
\] (4.16)

where,

\[L'_e = L^*_e + \frac{I_{xz}}{I_x} N^*_e\] \[N'_e = N^*_e + \frac{I_{xz}}{I_z} L^*_e\] \[L^*_e = \frac{L_e}{1 - I_{xz}(I_x I_z)^{-1}}\]

### 4.1.4 Interaction Matrix Linearisation

A classic solution to the IBVS control problem considers regulating the output error function, \(e = s(t) - s^*\), such that the system achieves an exponential decrease in error, \(e = \exp(-\lambda t)\). A desired velocity screw, \(T_{c}^*\), that will servo the camera towards the desired set of features, \(s^*\), and thus the goal pose, can then be derived from Equation (4.3), recognising that \(\dot{e} = -\lambda e = \dot{s}\),

\[T_{c}^* = -\lambda L_{s}^+(s(t) - s^*)\] (4.17)

Computing the inverse of the interaction matrix, \(L_{s}^+\), is however complicated given the function varies as a function of feature parameters. For this reason the interaction matrix is commonly approximated at its final value, i.e. calculated
given the desired set of feature parameters $L_{s=s^*}$, [165,187,189]. If the number of features is equal to the camera’s degrees of freedom such that, $k = m$, then a unique solution is available in the form of $L_{s}^{-1} = L_{s}^{-1}$. If however $k \geq m$ then the problem becomes one of finding the pseudo-inverse, $L_{s}^{+} = (L_{s}^{T}L_{s})^{-1}L_{s}^{T}$, where measurement noise is likely to result in an inconsistent set of equations for which the pseudo-inverse provides a form of least-squares solution [217].

In the case of a single line feature, only 2 feature parameters are available compared to the 6 DOF of the aircraft. In this instance, certain components of the object velocity can not be observed, a problem that is typically addressed through the inclusion of complementary sensors [186,187,189] or through the augmentation of the system model to include the vector of visual signals [185]. In this work, a solution is sought through the linearisation of the interaction matrix such that the vector of visual signals can be augmented with the aircraft dynamics model such that feature pose becomes a state of the overall system.

Linearising the interaction matrix begins with approximating motion in the image plane as small perturbations about a steady state condition, that is expressed here as,

$$T_e \triangleq T_{e_0} + t_e$$  \hspace{1cm} $$\Theta_l \triangleq \Theta_{l_0} + \theta_l$$ (4.18)

where $(t_e, \theta_l)$ represent perturbed motion about a reference position $(T_{e_0}, \Theta_{l_0})$. Terms $K_a$, $K_b$, $K_c$ and $K_d$ from (4.12) are likewise approximated as,

$$K_x \triangleq K_{x_0} + k_x$$ (4.19)

For term $K_a$, substituting the approximations of (4.18) results in,

$$K_a = \frac{1}{d_i}\left((a_i(\sin \Theta_{l_0} + \theta_l \cos \Theta_{l_0}) - b_i(\cos \Theta_{l_0} - \theta_l \sin \Theta_{l_0})) (T_{e_0} + t_e) - c_i f \right)$$

setting all perturbations to zero then yields $K_{a_0}$,

$$K_{a_0} = (T_{e_0}(a_i \sin \Theta_{l_0} - b_i \cos \Theta_{l_0}) - c_i f) / d_i$$

where $k_a$ can then be calculated as $k_a = K_a - K_{a_0}$, giving,

$$k_a = \frac{a_i \sin \Theta_{l_0} - b_i \cos \Theta_{l_0}}{d_i} t_e + \frac{T_{e_0}(a_i \cos \Theta_{l_0} + b_i \sin \Theta_{l_0})}{d_i} \theta_l$$ (4.20)
Similarly for terms $K_b$, $K_c$ and $K_d$,

$$
K_b \Rightarrow K_{b_0} = \left( T_{e_0}^2 + f^2 \right) / f \quad k_b = \left( 2T_{e_0} / f \right) t_e
$$

$$
K_c \Rightarrow K_{c_0} = \left( a_i \cos \Theta_{t_0} + b_i \sin \Theta_{t_0} \right) / d_i \quad k_c = \left( \left( b_i \cos \Theta_{t_0} - a_i \sin \Theta_{t_0} \right) / d_i \right) \theta_t
$$

$$
K_d \Rightarrow K_{d_0} = T_{e_0} / f \quad k_d = \left( 1 / f \right) t_e
$$

Interaction matrix term $T_e$ of (4.12) then becomes,

$$
\frac{d}{dt} (T_{e_0} + t_e) = + (K_{a_0} + k_a) (\sin \Theta_{t_0} + \theta_t \cos \Theta_{t_0}) (U_0 + u) \ldots
$$

$$
- (K_{a_0} + k_a) (\cos \Theta_{t_0} - \theta_t \sin \Theta_{t_0}) (V_0 + v) \ldots
$$

$$
+ (K_{a_0} + k_a) (K_{d_0} + k_d) (W_0 + w) \ldots
$$

$$
+ (K_{b_0} + k_b) (\cos \Theta_{t_0} - \theta_t \sin \Theta_{t_0}) (P_0 + p) \ldots
$$

$$
+ (K_{b_0} + k_b) (\sin \Theta_{t_0} + \theta_t \cos \Theta_{t_0}) (Q_0 + q)
$$

Recalling trim conditions define \{V_0, W_0, P_0, Q_0\} = 0, U_0 = V_P^1, while products of perturbations are assumed negligible ($\alpha^2 \approx 0, \alpha \beta \approx 0$),

$$
\dot{t}_e = + K_{a_0} \sin \Theta_{t_0} u - K_{a_0} \cos \Theta_{t_0} v + K_{a_0} K_{d_0} w \ldots
$$

$$
+ K_{b_0} \sin \Theta_{t_0} p + K_{b_0} \sin \Theta_{t_0} q + k_a, V_P \sin \Theta_{t_0} t_e \ldots
$$

$$
+ V_P (K_{a_0} \cos \Theta_{t_0} + k_{a_2} \sin \Theta_{t_0}) \theta_t + K_{a_0} V_P \sin \Theta_{t_0}
$$

(4.21)

where $k_{a_1}$ and $k_{a_2}$ are taken from (4.20) such that $k_a = k_{a_1} t_e + k_{a_2} \theta_t$.

Likewise, interaction matrix term $\dot{\Theta}_t$ of (4.12) then becomes,

$$
\frac{d}{dt} (\Theta_{t_0} + \theta_t) = - (K_{c_0} + k_c) (\sin \Theta_{t_0} + \theta_t \cos \Theta_{t_0}) (U_0 + u) \ldots
$$

$$
+ (K_{c_0} + k_c) (\cos \Theta_{t_0} - \theta_t \sin \Theta_{t_0}) (V_0 + v) \ldots
$$

$$
- (K_{c_0} + k_c) (K_{d_0} + k_d) (W_0 + w) \ldots
$$

$$
+ (K_{d_0} + k_d) (\sin \Theta_{t_0} + \theta_t \cos \Theta_{t_0}) (P_0 + p) \ldots
$$

$$
- (K_{d_0} + k_d) (\cos \Theta_{t_0} - \theta_t \sin \Theta_{t_0}) (Q_0 + q) \ldots
$$

$$
- (R_0 + r)
$$

\(^1\text{Airspeed is defined as } V_P = |V_0|, \text{ where } V_0 \text{ is the free stream velocity, not to be confused with the lateral component of steady state velocity, } V_0.\)
Thus perturbations of $\Theta_l$ can be expressed as,

$$\dot{\theta}_l = -K_{c0} \sin \Theta_{t_0} u + K_{c0} \cos \Theta_{t_0} v - K_{d0} w \ldots + K_{d0} \sin \Theta_{t_0} p - K_{d0} \cos \Theta_{t_0} q - r \ldots - V_P (K_{c0} \cos \Theta_{t_0} + k_{c1} \sin \Theta_{t_0}) \dot{\theta}_l - K_{c0} V_P \sin \Theta_{t_0} \quad (4.22)$$

Finally the linearised interaction matrix can be constructed from (4.21) and (4.22) resulting in,

$$\begin{bmatrix} \dot{t}_e \\ \dot{\theta}_l \end{bmatrix} = \begin{bmatrix} K_{a0} \sin \Theta_{t_0} & -K_{c0} \sin \Theta_{t_0} \\ -K_{a0} \cos \Theta_{t_0} & K_{c0} \cos \Theta_{t_0} \\ K_{a0} K_{d0} & -K_{a0} K_{d0} \\ K_{b0} \cos \Theta_{t_0} & K_{d0} \sin \Theta_{t_0} \\ K_{b0} \sin \Theta_{t_0} & -K_{d0} \cos \Theta_{t_0} \\ 0 & 0 \\ k_{a1} V_P \sin \Theta_{t_0} & 0 \\ a_{18} & a_{28} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \\ t_e \\ \theta_l \end{bmatrix} + \begin{bmatrix} K_{a0} V_P \sin \Theta_{t_0} \\ -K_{c0} V_P \sin \Theta_{t_0} \end{bmatrix} \quad (4.23)$$

where terms $a_{18}$ and $a_{28}$ are given by,

$$a_{18} = V_P (K_{a0} \cos \Theta_{t_0} + k_{a2} \sin \Theta_{t_0}) \quad a_{28} = -V_P (K_{c0} \cos \Theta_{t_0} + k_{c1} \sin \Theta_{t_0})$$

While the above relates perturbed motion of the aircraft to motion in the image plane, there still exists dependence on $a_i$, $b_i$, $c_i$ and $d_i$. Certain aspects of the feature’s geometry can however be deduced given the trim condition under which the aircraft model was linearised. Given a straight and level flight condition, the line feature can be assumed to lie in a plane parallel to the $(ox_b, oy_b)$ body axes of the aircraft at a distance $h$ below the aircraft, as depicted in Figure 4.4. Such a plane can be described by $z = h$, or $a = b = 0, c = 1, d = -h$, where $h$ is the aircraft height above the feature. Equation (4.23) then simplifies to,

$$\begin{bmatrix} \dot{t}_e \\ \dot{\theta}_l \end{bmatrix} = \begin{bmatrix} K_{a0} \sin \Theta_{t_0} & K_{a0} K_{d0} & K_{b0} \sin \Theta_{t_0} \\ 0 & 0 & -K_{d0} \cos \Theta_{t_0} \end{bmatrix} \begin{bmatrix} u \\ w \\ q \end{bmatrix} \ldots$$
where,

\[
K_{a0} = \frac{f}{h} \quad K_{b0} = \frac{T_{e0}^2 + f^2}{f} \quad K_{d0} = \frac{T_{e0}}{f}
\]
While $h$ is unknown, an approximation is made based on the desired height of inspection minus the standard height of the power poles in the inspection area. Finally, the reference position of the feature about which motion will be assumed to take place can be selected as the desired view during steady state tracking. This is chosen with the feature centred and vertical in the image plane ($T_e = 0, \Theta_l = 0$) reflecting the desired case when tracking in the absence of wind, which will be considered in Chapter 5. Substituting into (4.24) and removing zero columns results in,

$$\dot{x}_i = A_{i,1}x_a + A_{i,2}x_i$$

(4.25)

where,

$$A_{i,1} = \begin{bmatrix} -\frac{f}{h} & f & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \quad x_a = \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix}, \quad A_{i,2} = \begin{bmatrix} 0 & \frac{fV_P}{h} \\ 0 & 0 \end{bmatrix}, \quad x_i = \begin{bmatrix} t_e \\ \theta_l \end{bmatrix}$$

### 4.1.5 Full State Model

Under such conditions it is seen that longitudinal motion of the aircraft no longer influences motion of the feature in the image plane. Considering aircraft dynamics are also decoupled under the given trim condition, control of feature position should then be possible through lateral control alone, where longitudinal control can focus on maintaining airspeed and altitude, or in a practical sense, left under the control of an autopilot that would be performing such operations leading to visual tracking. Dynamics of a fixed wing air vehicle were developed in Section A.2, where a state representation for decoupled lateral equations of motion were shown to be expressed by (4.16), re-written here as,

$$\dot{x}_a = A_ax_a + B_au$$

(4.26)

where,

$$A_a = \begin{bmatrix} Y_v & 0 & -V_P & g \\ L'_v & L'_p & L'_r & 0 \\ N'_v & N'_p & N'_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad B_a = \begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L'_{\delta_a} & L'_{\delta_r} \\ N'_{\delta_a} & N'_{\delta_r} \\ 0 & 0 \end{bmatrix}, \quad u = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$
Control surface dynamics are modelled as first order systems on account of delays that would be introduced through the use of servo motors, where the transfer function can be expressed as,

\[
\frac{\delta(s)}{\delta_c(s)} = \frac{1}{\tau s + 1}
\]

(4.27)

with \(\delta_c\) and \(\delta\) the commanded and actual response of the control surface respectively, and \(\tau\) the servomotor time constant. Expressing this transfer function in state space form,

\[
\dot{u} = B_{c,1}u + B_{c,2}u_c
\]

(4.28)

with,

\[
B_{c,1} = \begin{bmatrix} -1/\tau_a & 0 \\ 0 & -1/\tau_r \end{bmatrix} \quad B_{c,2} = \begin{bmatrix} 1/\tau_a & 0 \\ 0 & 1/\tau_r \end{bmatrix} \quad u_c = \begin{bmatrix} \delta_{ac} \\ \delta_{rc} \end{bmatrix}
\]

where subscript \((a)\) and \((r)\) refer to aileron and rudder respectively.

One final addition is an integral term \(t_e(I) = \int t_e \, dt\) to Equation 4.25 in order to reduce steady state error during tracking, and is included in the state model with the augmentation of \(A_{1,2}\) and \(x_1\), now given by,

\[
A_{1,2} = \begin{bmatrix} 0 & 0 & \frac{f V_P}{F} \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad x_1 = \begin{bmatrix} t_e \\ t_e(I) \\ \theta_l \end{bmatrix}
\]

The full system model used to design the IBVS controller is then created by augmenting the aircraft state space model of (4.26) with the interaction matrix of (4.25) and control dynamics of (4.28), resulting (4.29). In this way, inner loop aircraft stabilisation and outer loop guidance control are integrated, allowing both platform stabilisation and feature tracking to be achieved simultaneously. While guidance loops in general provide reference commands to the inner stabilisation loop and hence may operate at a reduced rate [22], the guidance objective in this instance is met through visual control of a feature whose motion is a direct result of aircraft motion. As a result, the guidance loop must operate in conjunction with the aircraft’s inner control loop, which is facilitated through the
augmentation of the aircraft model and interaction matrix.

\[
\begin{bmatrix}
\dot{v} \\
\dot{p} \\
\dot{r} \\
\dot{\phi} \\
\dot{\theta} \\
\dot{\delta}_a \\
\dot{\delta}_r
\end{bmatrix}
= 
\begin{bmatrix}
Y_v & 0 & -V_p & g & 0 & 0 & 0 & Y_{\delta_a} & Y_{\delta_r} \\
L_v' & L_p' & L_r' & 0 & 0 & 0 & 0 & L_{\delta_a}' & L_{\delta_r}' \\
N_v' & N_p' & N_r' & 0 & 0 & 0 & 0 & N_{\delta_a}' & N_{\delta_r}' \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-f/h & f & 0 & 0 & 0 & 0 & fV_p/h & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/\tau_a & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/\tau_r
\end{bmatrix}
\begin{bmatrix}
v \\
p \\
r \\
\phi \\
\theta \\
\delta_a \\
\delta_r
\end{bmatrix}
\]

\[
= 
\begin{bmatrix}
0 \\
\vdots \\
1/\tau_a \\
0 \\
1/\tau_r
\end{bmatrix}
\begin{bmatrix}
\delta_{ac} \\
\delta_{rc}
\end{bmatrix}
\]

(4.29)

4.2 Control Design

Control is developed assuming access to full state feedback for which an optimal Linear Quadratic Regulator (LQR) control solution can be developed. Under such a design, closed loop feedback is provided through state feedback gain, \(K\), in the form of,

\[
\mathbf{u} = -K(\mathbf{x} - \mathbf{x}_{des})
\]

where \(K\) is derived in such a way to minimise the cost function,

\[
J = \int_0^{\infty} x^TQx + \rho u^TRu \, dt
\]

where both \(Q\) and \(R\) are symmetric positive definite matrices selected to provide appropriate weighting to meet control objectives, and \(\rho\) a positive constant that allows a trade off between control effort and the decrease in controlled output. Gain \(K\) is then calculated from the following [218],

\[
K = -R^{-1}B^TP
\]
where $\mathbf{P}$ is the unique positive semi-definite solution of the *Algebraic Riccati Equation* (ARE) given by,

$$0 = \mathbf{P}\mathbf{A} + \mathbf{A}^T\mathbf{P} + \mathbf{Q} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}$$

where the closed loop system is then given by,

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{BK})\mathbf{x} + \mathbf{B}\mathbf{x}_{\text{des}}$$

The closed loop system is then asymptotically stable given the system is both controllable and observable, where a system is said to be controllable if,

$$\text{rank} \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \cdots & \mathbf{A}^{n-1}\mathbf{B} \end{bmatrix} = n$$

and observable given,

$$\text{rank} \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{C}^{n-1}\mathbf{A} \end{bmatrix} = n$$

Using the system model of (4.29) with aircraft parameters, operating conditions and camera parameters as defined in Section 4.3, the system is found to be both controllable and observable thus ensuring the closed loop system is asymptotically stable given $\mathbf{Q}$ and $\mathbf{R}$ are symmetric positive definite matrices.

Given Sensor Track Error ($T_e$) is the primary state to minimise, term $\mathbf{Q}$ is initially constructed around a single non-zero value term at $Q_{5,5}$, with equal weight assigned to both control inputs with $\mathbf{R} = \mathbf{I}(2)$. In doing so, the optimal LQR problem reduces to,

$$J = \int_0^\infty t_e^2 + \rho (\delta_{a_e}^2 + \delta_{r_e}^2) \, dt \quad (4.30)$$

where it can be seen that control design is now one of finding the value of $\rho$ that gives the desired balance in performance and control input power. Figure 4.5 shows the effect of varying $\rho$ for $\rho_n > \rho_{(n+1)}$, with the controller applied to the Linear Time Invariant (LTI) model of (4.29) using aircraft parameters, operating

---

iiWhere $Q_{i,j}$ refers to the element of $\mathbf{Q}$ at row $i$, column $j$. 
conditions and camera parameters as defined in Section 4.3. Sensor Track Error, $T_e$, as measured in the image plane is converted from its original value in pixels to a percentage of frame width with respect to image centre to provide a relative measure of feature position, e.g. $T_e = 50\%$ infers track error of 512 pixels for a $2048 \times 1536$ pixel sensor. This is used in place of pixel values, as is processed by the feature extraction algorithm, as the pixel value can be less intuitive to visualise given it must be considered in relation to the sensor’s overall size.

As expected, control response of $T_e$ is seen to improve at the expense of increased control effort from both ailerons and rudder, with $\rho_1$ requiring far less control effort compared to that of $\rho_4$ that commands considerable deflection of both ailerons and rudder in achieving a fast response. Of particular interest however is the variation in manoeuvre that the controller is seen to perform...
as ρ decreases. From the plot of Bank Angle it is evident that controllers ρ₁ and ρ₂ bank the aircraft away from the feature, hence the initial increase in Tₑ, before centring the feature, or in practical terms, the response expected from performing a BTT manoeuvre. On the other hand, ρ₃ and ρ₄ are seen to bank towards the feature, allowing the feature to be centred almost immediately.

The practical implication however of banking downward facing sensors towards the feature is that the aircraft’s heading will be altered such that it will fly away from the feature, only requiring the aircraft to bank further, leading to an unstable condition. In the instance of ρ₃ and ρ₄ however, the controllers are seen to avoid this situation by applying opposite rudder, not only to the extent of counteracting the lateral force of the induced bank, but sufficient to alter the aircraft’s course towards the feature. As the aircraft approaches the feature, demand on bank angle required to view the feature is reduced, thus requiring less compensation from rudder, up until the point where the aircraft finally returns to wings level flight. A similar manoeuvre is performed in general aviation and is known as a Forward Slip, although rather than used to alter heading, is utilised to increase drag during landings where the manoeuvre can equally be used to maintain course over ground while flying at a sideslip [211, Chapter 8].

While the overall objective of the control task lies in centring the feature from the perspective of the inspection sensor FOV, of equal importance is ensuring data quality as the feature moves within the image plane is preserved, a condition not necessarily fulfilled by the above responses. To address this, a new term is introduced through Q₄₄ to minimise roll commanded during tracking. Figure 4.6 shows the results of adding such a term for two separate tunings, STT1 and STT2, with Q₄₄ and Q₅₅ set to 100 and 0.05, and 400 and 0.22 for each respectively, resulting in optimal state feedback gain matrices,

\[
K_{\{STT1\}} = \begin{bmatrix}
0.11 & -0.33 & -0.53 & -9.9 & -0.020 & 0.026 & 4.2 & 0.72 \\
-0.070 & -0.015 & -0.22 & -2.2 & 0.22 & 2.2 & 0.72 & 0.90
\end{bmatrix}
\]

\[
K_{\{STT2\}} = \begin{bmatrix}
0.17 & -0.65 & -0.70 & -20. & -0.050 & 0.0099 & 6.3 & 0.73 \\
-0.11 & -0.016 & -0.24 & -3.5 & 0.47 & 3.4 & 0.73 & 0.97
\end{bmatrix}
\]

The controller is now seen to perform Skid-to-Turn manoeuvres similar to those used in Chapter 3, maintaining wings-level flight while altering heading through rudder. Response of the first tuning, STT1, shows similar levels of
4.2 Control Design

Figure 4.6: Control response with the addition of term $Q_{4,4}$ to minimise bank angle and thus motion of the image feature. Controller is seen to utilise Skid-to-Turn manoeuvres under these conditions.

Figure 4.6: Control response with the addition of term $Q_{4,4}$ to minimise bank angle and thus motion of the image feature. Controller is seen to utilise Skid-to-Turn manoeuvres under these conditions.

The second controller, STT2, demonstrates the performance that can be achieved with appropriate tuning and sufficient control power. The feature is now seen to move smoothly into the image centre in almost half the time of STT1. As may be expected, a trade off is seen between decreasing the time taken to recentre the feature and the amount of motion seen by the sensor. As the response increases, the aircraft is commanded to enter greater angles of sideslip...
to generate increased turn rates from the STT manoeuvre, which from the perspective of the sensor, results in rotation about the principal axis. The resulting motion is similar to twisting the sensor and is far less intrusive compared to roll induced by BTT manoeuvres that subjects the sensor to a swinging motion.

While maintaining wings-level flight during tracking reduces motion and allows the inspection sensor to remain horizontal, it does limit the rate at which the feature can be recentred as it requires the aircraft to generate lateral motion. Where the inspection sensor can tolerate rotation, improved performance may be sought by taking advantage of the Forward-Slip manoeuvre that was observed by $\rho_3$ and $\rho_4$ in Figure 4.5. The ever present issue of commanding bank however is the risk of inducing unwanted motion in the form of the swinging or panning motion that, applied too fast, can lead to reduced data quality. Reducing the
rate at which roll is commanded can alleviate the issue and is incorporated in the design here through the addition of term $Q_{2,2}$ to minimise roll rate. Introducing a term to minimise yaw rate is also found to improve response and is incorporated through the addition of term $Q_{3,3}$. Figure 4.7 demonstrates two such designs, FS1 and FS2, where terms $Q_{2,2}$, $Q_{3,3}$ and $Q_{5,5}$ are set to 1, 1 and 0.1, and 5, 0.5 and 1 respectively, resulting in optimal state feedback gain matrices,

$$K_{(FS1)} = \begin{bmatrix}
0.10 & -0.30 & -0.36 & -0.75 & -0.17 & 0.53 & 4.0 & 0.24 \\
-0.15 & 0.056 & -0.77 & -2.02 & 0.27 & 2.8 & 0.24 & 2.5
\end{bmatrix} \quad (4.33)$$

$$K_{(FS2)} = \begin{bmatrix}
0.25 & -0.99 & -0.75 & -0.013 & -0.94 & -0.87 & 7.9 & 0.55 \\
-0.15 & 0.018 & -0.60 & -2.5 & 0.35 & 3.5 & 0.55 & 2.1
\end{bmatrix} \quad (4.34)$$

The response of the Forward-Slip designs are seen to provide improvements over the Skid-to-Turn designs of Figure 4.6, with faster convergence at reduced control effort. This is particularly evident through a comparison of STT2 and FS1, where a similar time to recentre the feature is seen, while FS1 uses far less control effort, to the extent that it commands similar levels to that of the slower response of STT1. Likewise, using similar levels of control effort, FS2 is seen to improve on STT2, recentring the feature in almost 2 s compared to 5 s for STT2.

### 4.2.1 State Estimation

A practical consideration not made up until this point is the accessibility of state measurements. Although an autopilot and vision system are assumed available to provide aircraft state information and feature parameters respectively, this information is likely to contain uncertainties including noise, offsets, quantisation errors and varying sampling rates. One issue with augmenting feature parameters with state information is the varying rates that can be expected between these measurements; where a reduced rate can be expected from feature extraction compared to the update rate of measurements available from an autopilot.

To address these issues, a state estimator is introduced to the system, with the overall system now expressed with the inclusion of measurement noise, $v$, and process noise, $w$,

$$\dot{x} = Ax + Bu + w \quad \quad y = Cx + v$$
where a Discrete Time Linear Kalman filter is introduced to the system in the form of [219],

\[
\begin{align*}
\mathbf{P}_k^- &= \mathbf{F}\mathbf{P}_{k-1}^+ \mathbf{F}^T + \mathbf{Q} \\
\mathbf{K}_k &= \mathbf{P}_k^- \mathbf{H}^T (\mathbf{H}\mathbf{P}_k^- \mathbf{H}^T + \mathbf{R})^{-1} \\
\hat{\mathbf{x}}_k^- &= \mathbf{F}\hat{\mathbf{x}}_{k-1}^+ + \mathbf{G}\mathbf{u}_{k-1} \\
\hat{\mathbf{x}}_k^+ &= \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{y}_k - \mathbf{H}\hat{\mathbf{x}}_k^-) \\
\mathbf{P}_k^+ &= (\mathbf{I} - \mathbf{K}_k\mathbf{H})\mathbf{P}_k^- (\mathbf{I} - \mathbf{K}_k\mathbf{H})^T + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^T
\end{align*}
\]

where \( \mathbf{F}, \mathbf{G}, \mathbf{H} \) represent the discretized state space model of the continuous system model (Equation 4.29), expressed in the form, \( \mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{G}\mathbf{u}_{k-1} + \mathbf{w}_{k-1}, \mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k; \) with process noise and measurement noise both modelled on zero mean multivariate normal distributions with covariance \( \mathbf{Q} \) and \( \mathbf{R} \) respectively; and \( (\mathbf{P}_k^-, \mathbf{P}_k^+) \) and \( (\hat{\mathbf{x}}_k^-, \hat{\mathbf{x}}_k^+) \) the a priori and a posteriori covariance and state estimates respectively.

Control is then a function of states estimates,

\[
u = -\mathbf{K}(\hat{\mathbf{x}} - \mathbf{x}_{\text{des}})
\]

where the full system is illustrated in Figure 4.8. The reader is referred to Appendix B for further details on the Kalman filter and the implementation within the simulation environment.

**Figure 4.8:** Overview of the combined aircraft dynamics and interaction matrix model (System Model) as derived in Section 4.1.5 where closed loop control is achieved through full state feedback (LQR Control) using state estimates (Kalman Filter).
4.3 Test Cases

Performance of the proposed controller would be assessed in the simulation environment developed in MATLAB Simulink® that allowed both the simulation of aircraft motion and data capture from a downward facing camera, as discussed in Section 3.2 and detailed in Appendix B. Aircraft dynamics were once again modelled on a 6 DOF nonlinear model of an Aerosonde® UAV, while control surface deflections were restricted in both range and rate to reflect practical limitations, with range restricted to $-30^\circ \leq \delta_a, \delta_r \leq 30^\circ$ and rate limited to $-45^\circ/s \leq \dot{\delta}_a, \dot{\delta}_r \leq 45^\circ/s$. Synthetic imagery was again generated assuming a flat earth model, with power poles supporting a conductor at a height of 10 m, spaced at intervals of 100 m. Each of the four designs, Skid-to-Turn STT1, STT2 and Forward-Slip FS1, FS2, would be tuned around a desired flight condition and would utilise the same gains for each subsequent simulation.

Flight parameters were chosen to reflect realistic flight conditions under which the UAV would typically operate. An airspeed of 100 km/h (54 kn) was chosen given the results of Chapter 3 found improvements in performance when airspeed was increased from 70 km/h to 100 km/h, while an altitude of 50 m was selected such that the sensor would capture the full width of a 20 m power line corridor. The sensor itself was again modelled on an ideal pin hole camera, free of distortion, with a resolution of $2048 \times 1536$ pixels, coupled with a 5 mm lens that presents a 1/3” sensor ($4.80 \times 3.60$ mm) with an approximate horizontal angular FOV of $50^\circ$. With operating conditions defined, and aircraft and camera models selected, the full state space representation of the system given by Equation (4.29) could then be calculated.

Stability derivatives for the Aerosonde UAV are calculated under the given operating conditions, where the reader is referred to Appendix A.2.4 for further details. Terms for the first order systems used to model control surface dynamics, given by Equation (4.27), were selected with a servomotor time constant of 0.25 s to reflect the slower operation that can be expected from smaller servomotors used on UAVs, where typical time constants can be expected to fall in a range of 0.05 - 0.25 s according to Nelson [208, p. 293]. The resulting state space representation for the IBVS control system under a straight, wings-level, symmetric flight condition, with camera orientation aligned with the body axes, with the feature centred and vertical, is then expressed as (4.36).
The scenario under which the controller would be tested was designed to reflect that of an aircraft tracking a feature with constant Sensor Track Error, a situation likely to arise in the presence of relative positioning errors that lead to aircraft flying slightly offcentre of the feature. This was achieved by initialising the UAV on a course parallel to the feature with a Cross Track Error of 10 m, which given the relative height and FOV of the sensor, sees the feature with a Sensor Track Error of 50%.

\[
\begin{bmatrix}
\dot{v} \\
\dot{p} \\
\dot{r} \\
\dot{\phi} \\
\dot{t}_e \\
\dot{t}_e \\
\dot{\theta}_t \\
\dot{\delta}_a \\
\dot{\delta}_r
\end{bmatrix} =
\begin{bmatrix}
-5.8e^{-1} & 0 & -28 & 9.8 & 0 & 0 & 0 & -1.4 & 3.7 \\
-4.2 & -25 & 12 & 0 & 0 & 0 & -156 & -2.2 \\
-0.84 & -3.2 & -1.3 & 0 & 0 & 0 & -6.0 & -30 \\
0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.10 & 5.0 & 0 & 0 & 0 & 0 & 2.8 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -4.0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4.0
\end{bmatrix}
\begin{bmatrix}
v \\
p \\
r \\
\phi \\
t_e \\
t_e(I) \\
\theta_t \\
\delta_a \\
\delta_r
\end{bmatrix}
\]

+ \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
0 \\
4.0
\end{bmatrix}
\begin{bmatrix}
\delta_a \\
\delta_r
\end{bmatrix}
\] 

(4.36)

The simulation begins with a basic autopilot controller maintaining airspeed and altitude, while providing bearing hold such that the aircraft reaches a steady state flight condition before initiating visual control. At that point, the autopilot relinquishes control of both aileron and rudder to the IBVS controller, while retaining control of both elevator and throttle for longitudinal control. This architecture is replicated within the simulation environment utilising two independent PID loops, regulating altitude through elevator, and airspeed through throttle. Aircraft state information required by the IBVS controller is assumed available from the autopilot at a rate of 100 Hz, while feature extraction is assumed to take place at 10 Hz. All signals provided to the IBVS controller have zero mean.
Gaussian noise added to reflect practical measurement conditions.

Aside from the ideal test case, under which the controllers were developed, the four designs would also be tested under varying Airspeed and Altitude to assess robustness of the controller to varying conditions. The result is a total of 5 test cases, as detailed in Table 4.1, for which each of the 4 control designs would be simulated. Ideal wind conditions are initially assumed, with wind effects considered later in Chapter 5.

<table>
<thead>
<tr>
<th>Test Cases</th>
<th>Altitude (m)</th>
<th>Airspeed (km/h)</th>
<th>Windspeed (km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal</td>
<td>50</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>Airspeed Variation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>→ Decrease</td>
<td>50</td>
<td>80</td>
<td>0</td>
</tr>
<tr>
<td>→ Increase</td>
<td>50</td>
<td>120</td>
<td>0</td>
</tr>
<tr>
<td>Altitude Variation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>→ Decrease</td>
<td>40</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>→ Increase</td>
<td>60</td>
<td>100</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.1: Test cases to assess the performance of the proposed controller.

4.4 Results

Results for Skid-to-Turn controllers STT1 and STT2 operating under the ideal test case are shown in Figure 4.9. Plots are of known states during the simulation, while it should be noted that the controller operates from state estimates provided by the Kalman filter that processes measurements with simulated noise and reduced rate feature measurements. The response of controller STT1 is seen to be very similar to that of the ideal response using the LTI model as presented in Figure 4.6, recentring the feature in approximately 8 s whilst avoiding excessive motion by maintaining wings level flight. The second design, STT2, also shows similar response to the LTI simulation, recentring the feature in approximately 5 s, although is seen to introduce unwanted motion initially in the form of an oscillation of $T_o$.

On closer inspection, the cause of the oscillation is seen to be as a result of the controller commanding control deflections in excess of the imposed rate limits.
This disrupts the coordinated application of aileron and rudder that is necessary for a smooth transition into the sideslip configuration, leading to the unwanted motion of $T_e$. In this instance, the oscillation is only small and unlikely to effect data capture, however the example does serve to highlight limitations that are likely to hinder practical systems.

Aside from introducing unwanted motion, STT2 is also found to command an average of 83% more control effort (82% ailerons, 84% rudder) compared to STT1 in achieving the 3 s, or 60%, improvement in response. It should be noted that comparisons of control effort are calculated based on the integral of each control signal over the 12 s time period used to recentre the feature. Furthermore, STT2 commands maximum aileron and rudder deflections of approximately 23°.
and $-17^\circ$ respectively, while STT1 only commands $8^\circ$ and $-7^\circ$. Although this may appear a case against *Skid-to-Turn*, in a practical context the performance of STT1, that avoids unwanted motion, is likely to satisfy the requirements of inspection tasks. Rather, these findings serve to highlight the practical limitations that may be experienced when using such manoeuvres for the task of tracking.

If the inspection task does call for a faster response and can accommodate the sensor rotating from horizontal, then gain may be had through the adoption of *Forward-Slip* manoeuvres. Figure 4.10 shows the results of FS1 and FS2 compared to that of STT2.

![Graphs showing comparison between STT2, FS1, and FS2](image)

**Figure 4.10:** *Forward-Slip* control response to offcentre feature versus *Skid-to-Turn*. FS1 is seen to achieve similar performance to STT2 while using significantly less control effort, while FS2 halves the response time of STT2 using similar levels of control.

Although FS1 and STT2 are both seen to recentre the feature in approximately 5 s, the *Forward-Slip* controller is seen to use considerably less control...
effort in the process, with FS1 commanding maximum aileron and rudder deflec-
tions of $8^\circ$ and $-8^\circ$ respectively compared to $23^\circ$ and $-17^\circ$ of STT2, while
control effort of ailerons is reduced by 67%, with similar levels of rudder used by
both. Reduced control effort can be expected as the Forward-Slip control design
utilises body roll to direct onboard sensors toward the feature, as opposed to the
Skid-to-Turn controller that relies on sideslip to generate lateral motion of the
aircraft. Rolling the aircraft not only demands less control effort, but can be
applied gradually as the feature is re-centred as a direct result of rotation, further
reducing camera motion.

While centring the feature with the Forward-Slip manoeuvre is achieved with
relative ease, it does require the aircraft to fly at a constant angle of sideslip
that would inevitably lead to poor efficiency, an issue highlighted by [111]. The
controller is however seen to handle this issue, although indirectly, as it seeks to
minimise control effort. By entering a greater angle of sideslip the aircraft is able
to move towards the feature, which in turn reduces the angle of bank required to
centre the feature, in turn reducing control effort. This process continues until
control effort is minimised at which point the aircraft has returned to wings level
flight. FS1 can be seen to go through this process from the plot of bank angle in
Figure 4.10, where from 3 s onwards the aircraft is seen to slowly return to wings
level flight, with both bank angle and control surfaces simultaneously reducing
to zero before the aircraft settles at a steady state bank angle of $-1^\circ$.

As was found with the STT designs, performance of the FS controller is found
to peak as commanded deflections begin to exceed control surface limits. This is
seen in the case of FS2 shown in Figure 4.10, that in a similar fashion to STT2,
shows signs of exceeding control surface rate limits. Again, the coordinated ap-
plication of aileron and rudder that allows the aircraft to enter the sideslip con-
figuration smoothly is impeded, causing unwanted motion in the camera frame,
even if very minor in this instance. In terms of performance, FS2 is seen to re-
centre the feature in 3 s, a 40% increase in performance over STT2 and FS1, with
maximum deflections of $15^\circ$ and $-15^\circ$, while only requiring an average of 29%
more control effort (32% ailerons, 26% rudder) over that of FS1.

Concern may be raised over the suitability of the aircraft model in accurately
representing the true response of the UAV during the forward-slip manoeuvre.
While data that validates the chosen Aerosonde model for these particular ma-
noeuvres is not available, measures can be taken to ensure motion stays within
acceptable limits given the linear stability and control coefficients the model provides. These limits are discussed in Appendix B.1.1, where of most concern is the angle of sideslip induced by the manoeuvre. Figure 4.11 shows the angle of sideslip induced by both the FS1 and FS2 control designs, where FS1 is seen to stay well within the $\pm 15^\circ$ limits, while FS2 momentarily exceeds the limit by $1^\circ$. For the purpose of demonstration that these simulations serve, it will be assumed that this discrepancy would not significantly impact the results.

![Sideslip Angle](image)

**Figure 4.11:** Sideslip induced during Forward-Slip manoeuvres.

The next series of simulations would test the control designs under variations in airspeed and altitude while maintaining control gains as set previously. Figure 4.12 shows the results of the four control designs operating at airspeeds of 80 km/h and 120 km/h versus the original airspeed of 100 km/h. For the STT controllers, the increase in airspeed is actually seen from Figures 4.12 (a, b) at 120 km/h to improve performance over the original, recentring the feature in less time while preserving a smooth transition. Although a decrease in performance can thus be expected as airspeed is lowered, Figures 4.12 (a, b) not only show the time to recentre at 80 km/h to increase but also introduce unwanted motion during the transition. This unwanted motion is particularly prevalent in the case of STT2 where the transition of $T_e$ is no longer smooth.

This can be once again accounted to commanded deflections exceeding rate limits that is only intensified as airspeed decreases, requiring larger deflections. The effect of airspeed on the Forward-Slip controllers is even more pronounced, in particular for FS1 where increased airspeed sees overshoot, while reduced airspeed results in steady state error that is slow to reduce. While FS2 is seen to handle
Figure 4.12: Performance of Skid-to-Turn (a, b) and Forward-Slip (c, d) control designs under varying Airspeeds of ±20 km/h (80 km/h, 100 km/h, 120 km/h).

the variations better than FS1, the transition is no longer as smooth.

Figure 4.13 shows the results of the four control designs operating at varying altitudes of ±10 m about the original design condition of 50 m. The effect on the sensor FOV is a scaling of $T_e$ such that at the same Cross Track Error of 10 m, $T_e$ is now observed at 68% and 40% for altitudes of 40 m and 60 m respectively. Higher altitudes are seen to slow the response slightly for each of the designs, although in general the impact is minimal. Decreases in altitude are seen to introduce overshoot to all the responses and in the cases of STT2 and FS2, increase the amount of unwanted motion slightly compared to the original. In general though, the effect of altitude is seen to be relatively minimal.
4.4 Results

A final observation worth noting is made through a comparison of Cross Track Error and Sensor Track Error. Figure 4.14 shows a comparison of the two metrics for both STT2 and FS2 control designs, where two clear issues are highlighted. The first issue, and most obvious, is a conflict of performance when assessing it in terms of centring the feature in the image plane versus recentring the aircraft over the feature. If one considers Cross Track Error, STT2 would appear the faster of the two controllers, recentring the aircraft over the feature in almost half the time of FS2, however clearly from the perspective of inspection, FS2 would be the better of the two, recentring the feature in the image plane in half the time of STT2.

---

Figure 4.13: Performance of Skid-to-Turn (a, b) and Forward-Slip (c, d) control designs as a result of varying Altitudes of ±10 m (40 m, 50 m, 60 m).
Figure 4.14: Example highlighting two issues of Cross Track Error; the first that FS2 would appear slower, while is actually faster recentring the feature, and second, the lack of insight Cross Track Error provides as to unwanted motion of the sensor.

The second, although more concerning issue, regards unwanted motion in the sensor FOV, or rather the inability to observe such motion when considering Cross Track Error. While both controllers are seen to display unwanted motion in one form or another, an analysis of Cross Track Error would reveal no such motion entering the sensor FOV, a key issue if such a controller is being considered for inspection purposes. These observations support the Results of Chapter 3 (Section 3.4), where an analysis of Cross Track Error failed to highlight the impact of BTT manoeuvres on captured imagery.

A summary of results for the Ideal test case is presented in Table 4.2, where Control Effort is expressed as percent increase in control effort relative to STT1.

<table>
<thead>
<tr>
<th>Control Design</th>
<th>Time to Centre (s)</th>
<th>Maximum Control (°)</th>
<th>Control Effort(^\text{iii}) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Aileron</td>
<td>Rudder</td>
</tr>
<tr>
<td>STT1</td>
<td>8</td>
<td>8</td>
<td>-7</td>
</tr>
<tr>
<td>STT2</td>
<td>5</td>
<td>23</td>
<td>-17</td>
</tr>
<tr>
<td>FS1</td>
<td>5</td>
<td>8</td>
<td>-8</td>
</tr>
<tr>
<td>FS2</td>
<td>3</td>
<td>15</td>
<td>-15</td>
</tr>
</tbody>
</table>

Table 4.2: Summary of results for control designs under ideal operating conditions.

\(^{iii}\)Relative to the Control Effort of STT1, where Control Effort is defined as the integral of each respective control signal.
4.5 Summary

This chapter has presented a novel IBVS control solution for the simultaneous tracking and data capture utilising Forward-Slip manoeuvres in place of Skid-to-Turn as was presented in Chapter 3. By augmenting the aircraft dynamic model with the interaction matrix of the image feature, a controller was developed that not only controlled the view of infrastructure but allowed motion that would otherwise degrade data quality, to be directly controlled and subsequently suppressed. While the Wings-Level Skid-to-Turn controller of Chapter 3 was also able to minimise motion in the image plane, the solution is indirect and restricts motion to wings-level flight. The Forward-Slip controller developed in this chapter not only offers a direct means to reduce motion but is also shown to increase performance, while reducing control effort, as long as the inspection sensor can tolerate rotation from horizontal during data collection.

Utilising similar levels of control effort, the Forward-Slip controller was shown to provide 67\% increase in performance over STT, reducing time to recentre from 5 s to 3 s. Likewise, achieving similar levels of performance between the two only required 67\% the control effort of ailerons by the FS controller, while the smooth deflection of control surfaces by the FS controller versus the sudden application required by the STT controller reduced maximum deflection of control surfaces by approximately 65\% and 53\% for aileron and rudder respectively. The FS control design was also found to be robust to both airspeed and height variations, although showed signs of unwanted motion as commanded deflections of control surfaces approached physical rate limits. This was found to be the main limiting factor, as restrictions of the control surfaces would impede the coordinated application of aileron and rudder require to ensure a smooth transition into the sideslip configuration. Although the issue was shown to be avoidable, it does serve to highlight the practical limitations of using such manoeuvres for the task of tracking.
In Chapter 4 a novel IBVS solution to the simultaneous tracking and data collection problem was proposed that enabled the UAV to utilise *Forward-Slip* manoeuvres in place of *Skid-to-Turn*, increasing performance while reducing overall control effort. The original development however considered the ideal case of no wind, for which the IBVS task can be generalised as one of centring the feature vertically within the image plane. With the introduction of wind, the course over ground made by the aircraft no longer aligns with aircraft heading and subsequently effects data capture as was discovered in Chapter 3, for which a solution was sought through minimisation of steady state *Sensor Track Error* through an integral based solution linked to desired line angle. Although the introduction of integral control can offer a sound solution in the presence of unknown wind, lag and overshoot are often associated with the response given the nature of compensation.

In this chapter, a solution is sought to improve performance of the *Forward-Slip* control design of Chapter 4 given an estimate of mean wind conditions are available. The solution sets about modelling the effects of wind on the interaction matrix that now observes both motion of the aircraft and relative motion of the
In addition to the effects of a constant wind disturbance, the chapter also seeks to investigate the controllers response in varying wind conditions including *Turbulence* that represents continuous fluctuations of the Earth’s atmosphere, and *Gusts* that represent larger discrete disturbances.

## 5.1 Problem Formulation

In the previous development of Chapter 4, motion of the camera relative to the ground based object was assumed to be as a direct result of aircraft motion alone. This allowed the camera velocity screw, $T_c = f\{V_c, \Omega_c\}$, that relates motion of the camera to perceived motion of an object, to be approximated from the aircraft states assuming the sensor was close to the aircraft c.g. and had relative alignment with the body axes. In the presence of wind however, there exists relative motion between the *Airmass* in which the aircraft flies and the *Earth* to which the object is fixed. Perceived motion of a ground based object as viewed from a sensor rigidly fixed to a moving platform, itself travelling in a moving frame, will thus be the combination of the two moving frames.

In terms of steady state tracking, the effect of wind is a variation between *Course Over Ground*, $\chi$, and *Aircraft Heading*, $\Psi$. Hence to achieve a desired ground track\(^1\), and thus allow continuous tracking along infrastructure, desired heading must be altered to compensate for wind. If wind is known, then a correction term can be calculated based on the *Wind Triangle* as illustrated in Figure 5.1 that shows the resultant ground velocity vector, $V_g$, as the summation of the aircraft velocity vector, $V_0$, and wind speed vector, $V_w$. The correction term, $\theta_{wc}$, can then be used to alter course to fly a desired ground track, and as such is more commonly known as a *Wind Correction Angle* (WCA). The resultant ground velocity vector can then be expressed as,

$$V_g = V_0 + V_w$$

---

\(^1\) *Course Over Ground* and *Ground Track* both refer to the path flown with respect to the fixed reference frame, although *Course Over Ground* will generally refer to path travelled, while *Ground Track* the path desired or currently being flown.
5.1 Problem Formulation

Figure 5.1: Course Over Ground, $\chi$, as a result of wind, $V_w$, and necessary Wind Correction Angle, $\theta_{wc}$, required to adjust aircraft heading, $\Psi$, such that the aircraft follows a Desired Ground Track.

and Earth-fixed frame requires velocity of the airmass to be expressed in aircraft body-fixed frame. This can be achieved using Euler Angles that express the orientation of the aircraft with respect to the reference frame in the form of a Direction Cosine Matrix\(^{ii}\) (DCM) given by,

$$^b R_e = \begin{bmatrix} C\Theta C\Psi & C\Theta S\Psi & -S\Theta \\ S\Phi S\Theta C\Psi - C\Phi S\Psi & S\Phi S\Theta S\Psi + C\Phi C\Psi & S\Phi C\Theta \\ C\Phi S\Theta C\Psi + S\Phi S\Psi & C\Phi S\Theta S\Psi - S\Phi C\Psi & C\Phi C\Theta \end{bmatrix} \quad (5.1)$$

where the notation $^y R_x$ is used to express the transformation of Frame ($x$) to Frame ($y$), so in this instance, from Earth-fixed reference frame ($e$) to the body-fixed frame ($b$).

This allows the velocity component of the camera velocity screw to be expressed as,

$$V_c = V_{ac} + \dot{^b R_e} V_w \quad (5.2)$$

while under the current wind model, rotation of the camera remains the sole contribution of aircraft rotation such that,

$$\Omega_c = \Omega_{ac} \quad (5.3)$$

Approximating aircraft motion as small perturbations about a steady state

\(^{ii}\)Note: Order of rotation is Yaw ($\Psi$), Pitch ($\Theta$), Roll ($\Phi$)
flight condition, as is approximated during the linearisation of the aircraft dynamic equations of motion (Section A.2), allows $\mathbf{V}_{ac}$ and $\mathbf{\Omega}_{ac}$ to be approximated by,

$$
\mathbf{V}_{ac} = \begin{bmatrix}
U_0 + u \\
V_0 + v \\
W_0 + w
\end{bmatrix}
\quad
\mathbf{\Omega}_{ac} = \begin{bmatrix}
P_0 + p \\
Q_0 + q \\
R_0 + r
\end{bmatrix}
$$

likewise, Euler Angles that express aircraft orientation with respect to the inertial reference frame are also approximated as,

$$
\Phi \triangleq \Phi_0 + \phi \\
\Theta \triangleq \Theta_0 + \theta \\
\Psi \triangleq \Psi_0 + \psi
$$

Recalling the Straight ($\dot{\Theta}_0$, $\dot{\Phi}_0$, $\dot{\Psi}_0 = 0$), Level ($\Phi_0$, $\gamma_0 = 0$), Symmetric ($V_0 = 0$) steady state trim condition about which the aircraft model is linearised (Appendix A.2.1), and adopting stability axes such that the body-fixed longitudinal axis, $\alpha b$, is rotated parallel to the free stream velocity vector, $\mathbf{V}_0$, then pitch angle, $\Theta_0$, and vertical velocity, $W_0$, are also zero, while longitudinal velocity, $U_0$, is then equal to airspeed, $V_P$, where $V_P = |\mathbf{V}_0|$. The DCM of Eqn. (5.1) can then be simplified using small angle approximations ($\cos \alpha \approx 1$, $\sin \alpha \approx \alpha$) and assuming products of perturbations are negligible ($\alpha^2 \approx 0$, $\alpha \beta \approx 0$), such that rotation between the two frames is approximated as,

$$
^{b}\mathbf{R}_e = \begin{bmatrix}
\cos \Psi_0 - \psi \sin \Psi_0 & \sin \Psi_0 + \psi \cos \Psi_0 & -\theta \\
-(\sin \Psi_0 + \psi \cos \Psi_0) & \cos \Psi_0 - \psi \sin \Psi_0 & \phi \\
\theta \cos \Psi_0 + \phi \sin \Psi_0 & \theta \sin \Psi_0 - \phi \cos \Psi_0 & 1
\end{bmatrix}
$$

When considering the effects of constant wind, it is common to neglect vertical contributions [82, 83, 116, 118], where their effects are generally associated with atmospheric disturbances arising from gusts and turbulence [208]. The resulting wind vector ($\mathbf{V}_w = V_w \angle \chi_w$) can then be expressed as $\mathbf{V}_w = \begin{bmatrix} V_{wx} & V_{wy} & 0 \end{bmatrix}^T$ from which the camera velocity screw can be calculated from (5.2, 5.3) to give,

$$
\mathbf{T}_c = \begin{bmatrix}
(V_P + K_{w_1} + u + K_{w_2} \psi) & (K_{w_2} + v - K_{w_1} \psi) & \cdots \\
(w + K_{w_1} \theta - K_{w_2} \phi) & p & q & r
\end{bmatrix}^T
$$
where,

\[
K_{w1} = V_{wx} \cos \Psi_0 + V_{wy} \sin \Psi_0 \\
K_{w2} = V_{wy} \cos \Psi_0 - V_{wx} \sin \Psi_0
\] (5.4)

As the camera velocity screw has been altered to that used in the original linearisation of the interaction matrix in Section 4.1.4, the linearisation must be repeated taking into account the new terms.

Recalling the interaction matrix of the line feature as detailed in Section 4.1.2, Equation (4.12),

\[
\begin{bmatrix}
\dot{T} \\
\dot{\Theta}
\end{bmatrix}
= 
\begin{bmatrix}
K_a S_{\Theta_l} & -K_a C_{\Theta_l} & K_a K_d & K_b C_{\Theta_l} & K_b S_{\Theta_l} & 0 \\
-K_c S_{\Theta_l} & K_c C_{\Theta_l} & -K_c K_d & K_d S_{\Theta_l} & -K_d C_{\Theta_l} & -1
\end{bmatrix}
T_e
\]

with,

\[
K_a = \frac{T_e (a_i \sin \Theta_l - b_i \cos \Theta_l) - c_i f}{d_i} \\
K_b = \frac{f^2 + T_e^2}{f} \\
K_c = \frac{a_i \cos \Theta_l + b_i \sin \Theta_l}{d_i} \\
K_d = \frac{T_e}{f}
\]

The interaction matrix is then linearised approximating feature motion as,

\[
T_e \triangleq T_{e0} + t_e \\
\Theta_l \triangleq \Theta_{l0} + \theta_l \\
K_x \triangleq K_{x0} + k_x
\] (5.5)

where \((t_e, \theta_l, k_x)\) represent perturbed motion about the respective reference position \((T_{e0}, \Theta_{l0}, K_{x0})\), where \(x \in \{a, b, c, d\}\). Terms \(K_x\) are then given by,

\[
K_a \Rightarrow K_{a0} = \frac{T_{e0} (a_i \sin \Theta_{l0} - b_i \cos \Theta_{l0}) - c_i f}{d_i} \\
k_a = k_{a1} t_e + k_{a2} \theta_l
\]

\[
K_b \Rightarrow K_{b0} = \frac{T_{e0}^2 + f^2}{f} \\
k_b = \left(\frac{2T_{e0}}{f}\right) t_e
\]

\[
K_c \Rightarrow K_{c0} = \frac{(a_i \cos \Theta_{l0} + b_i \sin \Theta_{l0})}{d_i} \\
k_c = k_{c1} \theta_l
\]

\[
K_d \Rightarrow K_{d0} = \frac{T_{e0}}{f} \\
k_d = \left(\frac{1}{f}\right) t_e
\]

with,

\[
k_{a1} = -k_{c1} = \frac{a_i \sin \Theta_{l0} - b_i \cos \Theta_{l0}}{d_i} \\
k_{a2} = \frac{T_{e0} (a_i \cos \Theta_{l0} + b_i \sin \Theta_{l0})}{d_i}
\]
Term $\dot{T}_e$ of the interaction matrix then becomes,

$$
\frac{d}{dt}(T_{e0} + t_e) = + (K_{a0} + k_a)(\sin \Theta_{l0} + \theta t \cos \Theta_{l0})(U_0 + K_{w1} + u + K_{w2} \psi) + \\
- (K_{a0} + k_a)(\cos \Theta_{l0} - \theta t \sin \Theta_{l0})(V_0 + K_{w2} + v - K_{w1} \psi) + \\
+ (K_{a0} + k_a)(K_{d0} + k_d)(W_0 + w + K_{w1} \theta - K_{w2} \phi) + \\
+ (K_{b0} + k_b)(\cos \Theta_{l0} - \theta t \sin \Theta_{l0})(P_0 + p) + \\
+ (K_{b0} + k_b)(\sin \Theta_{l0} + \theta t \cos \Theta_{l0})(Q_0 + q)
$$

Recalling that the steady straight, wings level, symmetric trim condition under which the aircraft model is linearised define $\{V_0, W_0, P_0, Q_0\} = 0$, $U_0 = V_p$ and that as part of linearisation and assuming small perturbations allows products of perturbations to be considered negligible such that $(\alpha^2 \approx 0, \alpha \beta \approx 0)$, the above becomes,

$$
i_e = + K_{a0} \sin \Theta_{l0} u - K_{a0} \cos \Theta_{l0} v + K_{a0} K_{d0} w + K_{b0} \cos \Theta_{l0} p + \\
+ K_{b0} \sin \Theta_{l0} q + K_{a0} K_{d0} K_{w1} \theta - K_{a0} K_{d0} K_{w2} \phi + \\
+ K_{a0} (K_{w1} \cos \Theta_{l0} + K_{w2} \sin \Theta_{l0}) \psi + \\
+ k_{a1} (V_P + K_{w1}) \sin \Theta_{l0} - K_{w2} \cos \Theta_{l0}) t_e + \\
+ [K_{w2} (K_{a0} \sin \Theta_{l0} - K_{w2} \cos \Theta_{l0}) + \\
+ (V_P + K_{w1}) (K_{a0} \cos \Theta_{l0} + k_{a2} \sin \Theta_{l0})] \theta t + \\
+ K_{a0} (V_P + K_{w1}) \sin \Theta_{l0} - K_{w2} \cos \Theta_{l0})
$$

(5.6)

Likewise, term $\dot{\Theta}_t$ of the interaction matrix becomes,

$$
\frac{d}{dt}(\Theta_{l0} + \theta t) = - (K_{c0} + k_c)(\sin \Theta_{l0} + \theta t \cos \Theta_{l0})(U_0 + K_{w1} + u + K_{w2} \psi) + \\
+ (K_{c0} + k_c)(\cos \Theta_{l0} - \theta t \sin \Theta_{l0})(V_0 + K_{w2} + v - K_{w1} \psi) + \\
- (K_{c0} + k_c)(K_{d0} + k_d)(W_0 + w + K_{w1} \theta - K_{w2} \phi) + \\
+ (K_{d0} + k_d)(\sin \Theta_{l0} + \theta t \cos \Theta_{l0})(P_0 + p) + \\
- (K_{d0} + k_d)(\cos \Theta_{l0} - \theta t \sin \Theta_{l0})(Q_0 + q) + \\
- (R_0 + r)
$$

---

$^\dagger$Airspeed is defined as $V_p = |V_0|$, where $V_0$ is the free stream velocity, not to be confused with steady state lateral velocity component $V_0$. 
where substitution of trimmed steady state conditions leads to,

\[
\dot{t}_l = -K_{c_0} \sin \Theta_{l_0} u + K_{c_0} \cos \Theta_{l_0} v - K_{c_0} K_{d_0} w + K_{d_0} \sin \Theta_{l_0} p \ldots \\
- K_{d_0} \cos \Theta_{l_0} q - r - K_{c_0} K_{d_0} K_{w_1} \theta + K_{c_0} K_{d_0} K_{w_2} \phi \ldots \\
- K_{c_0} (K_{w_1} \cos \Theta_{l_0} + K_{w_2} \sin \Theta_{l_0}) \psi \ldots \\
+ [K_{w_2} (k_{c_1} \cos \Theta_{l_0} - K_{c_0} \sin \Theta_{l_0}) \ldots \\
- (V_P + K_{w_1}) (K_{c_0} \cos \Theta_{l_0} + k_{c_1} \sin \Theta_{l_0}) ] \theta_l \ldots \\
+ K_{c_0} (K_{w_2} \cos \Theta_{l_0} - (V_P + K_{w_1}) \sin \Theta_{l_0}) 
\] (5.7)

At this point assumptions can be made regarding the plane in which the feature lies. The same approximations as were made in Section 4.1.4 can be applied here as the feature can still be assumed to lie in a plane horizontal to the body axes of the aircraft, as wings-level flight is still sought during steady state tracking. Given alignment between camera and body axes, the plane can be described as \( z = h \), or \( a = b = 0, c = 1, d = -h \), with \( h \) height above the feature.

Further simplification of the interaction matrix can be made by introducing the desired pose of the feature. Although Desired Line Angle will no longer be zero, the overall objective of the tracking controller remains the same, minimising \( T_e \) such that the feature appears in the centre of the sensor FOV such that \( (T_{e_0} = 0) \). Terms \( K_{x_0} \) and \( k_x \) then reduce to,

\[
K_{a_0} = \frac{f}{h} \quad K_{b_0} = f \\
\{ K_{c_0}, K_{d_0}, k_{a_1}, k_{a_2}, k_b, k_{c_1} \} = 0
\]

Substituting these values into Equations (5.6) and (5.7) then allows the interaction matrix to be formed as follows,

\[
\begin{bmatrix}
\dot{t}_l \\
\dot{\theta}_l \\
\dot{\psi} \\
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = 
\begin{bmatrix}
\frac{f}{h} S \Theta_{l_0} & -\frac{f}{h} C \Theta_{l_0} & K_{w_3} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} 
\begin{bmatrix}
u \\
v \\
\psi \\
p \\
q \\
r
\end{bmatrix}
+
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
t_e \\
\theta_l \\
\psi \\
p \\
q \\
r
\end{bmatrix}
+
\begin{bmatrix}
\frac{f}{h} (V_P + K_{w_1}) S \Theta_{l_0} - K_{w_2} C \Theta_{l_0} \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\] (5.8)
with $C\Theta_{lo}$ ($S\Theta_{lo}$) used to represent $\cos \Theta_{lo}$ ($\sin \Theta_{lo}$) respectively, and,

$$K_{w3} = \frac{f}{h} \left( K_{w1} \cos \Theta_{lo} + K_{w2} \sin \Theta_{lo} \right)$$

It can be seen that the inclusion of wind has introduced a constant term to $i_e$, therefore if Sensor Track Error, $T_e$, is to ever reach a steady state condition then $i_e = 0$, which only occurs if,

$$(V_P + K_{w1}) \sin \Theta_{lo} = K_{w2} \cos \Theta_{lo}$$

Recalling $K_{w1}$ and $K_{w2}$ from (5.4) allows the condition to be expressed in terms of steady state heading, $\Psi_0$, and wind velocity $(V_{wx}, V_{wy})$ as,

$$V_P S\Theta_{lo} = (V_{wy} C\Psi_0 - V_{wx} S\Psi_0) C\Theta_{lo} - (V_{wx} C\Psi_0 + V_{wy} S\Psi_0) S\Theta_{lo} \quad (5.9)$$

While aircraft heading, $\Psi$, can not be measured directly from the image plane,

---

iv It should be noted that $V_0$ is steady state lateral velocity, where lateral velocity is approximated as $V \triangleq V_0 + v$, not to be confused with freestream velocity vector $V_0$.  

**Figure 5.2:** Relationship between aircraft heading, $\Psi_0$, and desired ground track, $\chi_d$, during steady state tracking of the feature ($V_0 = 0 \Rightarrow \beta = 0$).
it can be seen from Figure 5.2 that during steady state tracking of the feature the relative angle between aircraft heading and desired course over ground, $\chi_d$, is equal to the orientation of the extracted feature, $\Theta_l$, thus desired pose of the feature is given by,

$$\Theta_{l0} = \chi_d - \Psi_0$$ (5.10)

Recognising that $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$ and $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$ leads to,

$$V_P S\Theta_{l0} = C^2 \Theta_{l0} (V_{wy} C \chi_d - V_{wx} S \chi_d) + S^2 (V_{wy} C \chi_d - V_{wx} S \chi_d)$$

Finally, recognising that $\cos^2 \alpha + \sin^2 \alpha = 1$,

$$V_P \sin \Theta_{l0} = (V_{wy} \cos \chi_d - V_{wx} \sin \chi_d)$$ (5.11)

If the wind is then expressed in terms of magnitude and direction, $\mathbf{V}_w = V_w \angle \chi_w$, with,

$$V_{wx} = V_w \cos \chi_w$$
$$V_{wy} = V_w \sin \chi_w$$

the relationship simplifies to,

$$V_P \sin \Theta_{l0} = V_w \sin(\chi_w - \chi_d)$$

However inspecting the vectors of Figure 5.1 it is seen that the Wind Correction Angle can be expressed as,

$$\theta_{wc} = \arcsin \left( \frac{|\mathbf{V}_w|}{|\mathbf{V}_0|} \sin (\angle \mathbf{V}_w - \angle \mathbf{V}_g) \right)$$

Thus during steady state tracking in the presence of wind, the desired line angle will equal the Wind Correction Angle, thus steady state feature parameter, $\Theta_{l0}$, becomes,

$$\Theta_{l0} = \theta_{wc}$$

Finally it is noted from (5.10) that perturbations of line angle orientation, $\theta_l$, will be as a direct result of perturbations of yaw angle, $\psi$, given that desired
ground track remains constant, such that $\theta_l = -\psi$. The interaction matrix of (5.8) then becomes,

$$
\begin{bmatrix}
\dot{t_c} \\
\dot{\theta}_l
\end{bmatrix} = 
\begin{bmatrix}
\int f S \theta_{wc} & f S \theta_{wc} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
u \\
q
\end{bmatrix} + 
\begin{bmatrix}
-f C \theta_{wc} & f C \theta_{wc} & 0 \\
0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
v \\
p \\
r
\end{bmatrix} + 
\begin{bmatrix}
0 & f \left(V_{pc}\theta_{wc}\right) \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
t_c \\
\theta_l
\end{bmatrix}
$$

(5.12)

While the presence of wind now sees $\dot{t}_c$ influenced by longitudinal motion perturbations $(u, q)$ the relationship is seen to be small given both are relative to $\sin \theta_{wc}$, where in general $\theta_{wc}$ will be small given $V_w \ll V_P$. Given longitudinal and lateral dynamics of the UAV are decoupled under the desired trim condition, it would appear counter-intuitive to re-couple the dynamics in order to utilise the small gain in feature control it would offer. For this reason, longitudinal motion is considered to remain constant and left under the control of an onboard autopilot maintaining airspeed via throttle and altitude via elevator. Control and state estimation are then performed in the same manner as developed in Section 4.2, to produce Forward-Slip control designs.

## 5.2 Atmospheric Disturbances

While the current development seeks to address the effects of constant wind on data collection during tracking, it is also important to consider the effects of atmospheric disturbances that will be inevitable given the task is performed in an outdoor environment. Although estimates of mean wind conditions can be made through a combination of ground observations and onboard measurements [220, 221], atmospheric disturbances in the form of Turbulence and Gusts occur randomly and will remain unknown to the controller. Assessing the response of the controller to such conditions is enabled through the adoption of appropriate disturbance models that are discussed in the following section.
5.2 Atmospheric Disturbances

5.2.1 Turbulence Model

Turbulence in the context of statistical modelling of the Earth’s atmosphere refers to the continuous, random fluctuations of the airmass generated by solar heating and various chemical, thermodynamic and electromagnetic processes. Given the random nature under which the disturbances are generated, modelling is generally performed using statistical methods, where common models include the von Karman and Dryden spectral forms [208].

In this work the von Karman model is selected for which the power spectral density for the turbulence velocities are given by [222],

\[ \Phi_{ug}(\Omega) = \sigma_{u}^2 \frac{2L_u}{\pi} \frac{1}{[1 + (1.339L_u\Omega)^2]^{5/6}} \]
\[ \Phi_{vg}(\Omega) = \sigma_{v}^2 \frac{2L_v}{\pi} \frac{1 + \frac{8}{3}(1.339L_v\Omega)^2}{[1 + (1.339L_u\Omega)^2]^{11/6}} \]
\[ \Phi_{wg}(\Omega) = \sigma_{w}^2 \frac{2L_w}{\pi} \frac{1 + \frac{8}{3}(1.339L_v\Omega)^2}{[1 + (1.339L_u\Omega)^2]^{11/6}} \]

where \( \sigma \) is the root-mean-square turbulence intensities and term \( L \) is the turbulence scale lengths. The turbulence model selected in this instance is based on the low-altitude disturbance model as specified in Military Specification MIL-F-8785C, Section 3.7.3.4 [222], for which pre-existing software within the Aerosim blockset can be utilised.

5.2.2 Discrete Gust Model

Where Turbulence characterises continuous fluctuations of the Earth’s atmosphere, Gusts refer to the discrete variations that are generally associated with larger disturbances. A very simple approximation of a Gust can be made through a step change in wind speed, whereby windspeed is assumed to change instantaneously resulting in a sharp gust, while a more realistic model that accounts for rise time can be sought through a one-minus-cosine “1-cosine” model, expressed by [223],

\[ V_g = \frac{1}{2} V_{gm} \left( 1 - \cos \frac{\pi x}{d_m} \right) \quad 0 \leq x \leq 2d_m \]
where $V_{gm}$ is the magnitude of the gust and $2 \times d_{m}$ the overall length of the gust. These can be further developed to include single ramp gusts at an arbitrary starting position, $x_1$,

$$V_g = \begin{cases} 
0 & d_1 < x_1 \\
\frac{1}{2} V_{gm} \left( 1 - \cos \frac{\pi (x - x_1)}{d_1} \right) & x_1 \leq x \leq x_1 + d_1 \\
V_{gm} & x > x_1 + d_1
\end{cases}$$

or combined to create a sequence of gusts [222],

$$V_g = \begin{cases} 
0 & d_1 < 0 \\
\frac{1}{2} V_{g1m} \left( 1 - \cos \frac{\pi x}{d_1} \right) & 0 \leq x \leq d_1 \\
V_{g1m} & d_1 < x < d_1 + d_s \\
V_{g1m} - \frac{1}{2} V_{g2m} \left( 1 - \cos \frac{\pi (x - d_{1,s})}{d_2} \right) & d_{1,s} \leq x \leq d_{1,s,2} \\
V_{g1m} - V_{g2m} & x > d_{1,s,2}
\end{cases}$$

where $d_{x,y} = d_x + d_y$. The sequence can then be continued to create any combination of gust disturbances.

### 5.3 Test Cases

To assess the performance of the proposed design, the controller would be simulated in the simulation environment developed in MATLAB Simulink®, as described in Appendix B, with the addition of models for Constant, Turbulent and Gusting wind conditions as detailed in Section B.1.3. Performance of the design would be assessed against two versions of the original FS1 controller as was developed in Chapter 4. The first design would remove integral control from the model to assess the effects of providing no means of compensation for wind, for which results are labelled No Compensation. The second design would consider the performance achieved through the inclusion of integral control of Sensor Track Error, $T_e$, that was present in the first development, although further tuned in this instance to reduce the effects of wind. The proposed controller was then developed based on the same tuning as FS1 to provide a fair comparison with the
original controllers. Operating conditions would remain the same as those used in the simulations performed in Section 4.3 to allow appropriate comparison of results, with airspeed set at 100 km/h (54 knots) and height above ground set at 50 m AGL. A sensor with a horizontal FOV of 50° was again selected, such that the sensor could capture data over the full width of a 20 m power line corridor.

Wind conditions would be simulated through a combination of Constant wind, known to the controller, and atmospheric disturbances in the form of Turbulence and Gusts, that remain unknown. Turbulence would be simulated in each of the simulations, utilising a von Karman turbulence model integrated into the aircraft model as part of the Aerosim blockset. Initial simulations would test the ability to recentre the feature in the sensor FOV in the presence of both 18 km/h (10 kn) and 28 km/h (15 kn) crosswinds with respect to the feature, representing a worst case scenario in terms of wind direction. The UAV would perform the manoeuvre from either side of the feature to test the performance of the controller flying ‘into’ and ‘with’, for which varying responses could be expected.

The second series of simulations would test the performance of the controller in the presence of isolated gusts, and would include a second variation of the Forward-Slip controller in the form of FS2 as was developed in Chapter 4, reformulated to include wind compensation. Gusts disturbances would varying in magnitude from 10% to 50% of the nominated wind speed, with gust lengths adjusted from 5 s to 25 s respectively. Mean wind speed was increased to reflect moderate-to-strong wind conditions of 37 km/h (20 kn), now acting at a bearing of 70° with respect to north, where the feature is aligned north-south, such that wind conditions produce components acting both along the feature at 13 km/h (7 kn) and across the feature at 35 km/h (19 kn). The final series of simulations would test the performance of the controller in the presence of longer, sustained Gusts, each lasting 50 s, with rise and fall times of 20 s. A combination of Gusts were developed to test a range of conditions, subjecting the aircraft to a rise in windspeed of 20%, followed by a fall of 10%, further reduced by 30%, finally returning to normal with a rise 20%, resulting in a 230 s disturbance. Throughout the simulations the controller would have access to mean wind conditions, i.e. non varying conditions, from which an estimate of the Wind Correction Angle would be calculated using desired airspeed as an approximation for true airspeed. The result is an approximation of the Wind Correction Angle as a constant value that would otherwise vary as a function of airspeed and relative wind.
5.4 Results

The first series of results presented in Figure 5.3 show the response of the three separate control designs in the presence of an 18 km/h (10 kn) wind, with the aircraft approaching from two locations such that it flies ‘with’ the wind in (a) and ‘into’ the wind in (b). Sensor Track Error, $T_e$, is again provided as a percentage of frame width from image centre to provide a measure independent of sensor resolution, for example, $T_e = 50\%$ infers sensor track error of 512 pixels for a $2048 \times 1536$ pixel sensor. *No Compensation* shows the response of the IBVS controller designed under the assumption of no wind and thus has desired line angle set to zero ($\Theta_{l0} = 0^\circ$). The response is seen to be far from ideal, with the controller overshooting and then failing to recentre the feature, resulting in a steady state tracking error of approximately 50%.

![Figure 5.3](image)

(a) Control response flying ‘with’ wind.  
(b) Control response flying ‘into’ wind.

**Figure 5.3:** Control response of control design FS1 flying in a 18 km/h (10 kn) cross wind with respect to the feature.

In addition, the controller is seen to maintain a *Forward-Slip* manoeuvre, as opposed to using it to return to level flight, that is evident from the constant bank angle.
angle and continued application of aileron and rudder. This can be explained as a result of conflicting goals, recentring the feature while the controller expects the feature to be vertical, a condition that can not be satisfied in the presence of a crosswind, without maintaining sideslip. The impact of such steady state tracking error on data collection is highlighted in Figure 5.4a that shows the simulated view of the feature during tracking. It should be noted that the FOVs of Figure 5.4 are taken from the simulation of Figure 5.3a, i.e. flying ‘with’ the wind, although is the same for Figure 5.3b, i.e. flying ‘into’ the wind, once the aircraft reaches steady state conditions, which is achieved by $t = 20\,\text{s}$.

The second response, Integral, adds integral control of Sensor Track Error to compensate for steady state error introduced by wind. This is seen to successfully recentre the feature in just over 10 s in both instances, although a large amount of overshoot is experienced when flying ‘with’ the wind of Figure 5.3b. Rather than returning to wings level flight, the controller is also seen to maintain a steady state bank angle of approximately $20^\circ$ which is not only inefficient, but from Figure 5.4b, can be seen to result in the feature being viewed at an angle, even if centred. The final response, Wind Correction, shows the response of the new
Mean Wind Compensation

control design that takes into account the Wind Correction Angle. The controller is not only seen to recentre the feature within 5 s with no overshoot, but is also seen to return to wings level flight ($\Psi = 0^\circ$) that not only provides an efficient flight condition but also provides an ideal view angle for the onboard inspection sensor, as evident from Figure 5.4c. Turbulence introduced to the longitudinal, lateral and vertical axes of the aircraft is shown in Figure 5.5.

![Turbulence Graph](image)

(a) Turbulence introduced to 18 km/h (10 kn) cross wind scenario.

![Turbulence Graph](image)

(b) Turbulence introduced to 28 km/h (15 kn) cross wind scenario.

Figure 5.5: Turbulence as introduced through the von Karman model to the 18 km/h (10 kn) cross wind scenario of Figure 5.3 and 28 km/h (15 kn) cross wind scenario of Figure 5.6.

The second series of results shown in Figure 5.6 compare the performance of the controllers in the presence of a stronger cross wind of 28 km/h (15 kn). The feature is almost seen to leave the field of view in both cases of No Compensation and although the corridor remains in view, as seen in Figure 5.4d, the angle at which it is observed is far from ideal. The response of the Integral controller is also seen to degrade, where the ‘with’ wind case of Figure 5.6 is now seen to introduce unwanted motion in the form of oscillations of the sensor as the controller initially
attempts to recentre the feature, although does manage to recover, recentring the feature after approximately 12 s.

Flying ‘into’ the wind is seen to be less of an issue, with the controller again recentring the feature in approximately 10 s. Control effort is seen to increase in both instances, while the angle at which the controller holds the UAV during steady state tracking is now almost 30°, angling the sensor even further during steady state tracking, as seen in Figure 5.4e.

![Graphs](image)

(a) Control response flying ‘with’ wind. (b) Control response flying ‘into’ wind.

**Figure 5.6:** Control response of control design FS1 flying in a 28 km/h (15 kn) cross wind with respect to the feature.

The response of *Wind Compensation* on the other hand is seen to be very similar to that of the 18 km/h (10 kn) test case, recentring the feature in 5 s in both instances, the only downside being a small amount of steady state error in the case of Figure 5.3a. The controller is also seen to use similar levels of control effort, with neither aileron nor rudder being commanded past 10°. The overall response is still smooth with no significant overshoot and as can be seen from Figure 5.4f maintains good position during steady state tracking.
The next series of simulations would test the controller with wind compensation when subjected to isolated Gust disturbances ranging from 10% of mean wind to 50%, where mean wind conditions are increased to 37 km/h (20 kn) acting at 70° relative to north, where the feature is aligned north-south. Figure 5.7a shows the results for FS1 and FS2 control designs, where the five gusts disturbances can be seen in plots North Wind and East Wind of Figure 5.7b.

(a) Control response of FS1 and FS2 over the 5 minute simulation.

(b) Gust and Turbulence introduced during the simulation.

Figure 5.7: Control response of FS1 and FS2 to isolated gusts varying from 10% to 50% of mean wind conditions set at 37 km/h (20 kn) acting at 70° relative to north.

While both control designs are seen to maintain the feature in the FOV during each of the five disturbances, FS2 is seen to outperform FS1, although could be expected given FS2 was tuned in Chapter 4 to provide a faster response in reducing Sensor Track Error compared to FS1. Even in the worst case scenario of the 50% gust, which represents a momentary increase in wind speed from 37 km/h (20 kn) to 55 km/h (30 kn), both controllers are seen to handle the situation effectively, with FS1 keeping the feature within 50% of the image centre, while FS2 manages to keep $T_e$ below 20%.
5.4 Results

Observing bank angle and control deflections of aileron and rudder it can be seen that the controller compensates for the disturbance by re-entering a Forward-Slip manoeuvre. This not only prevents the feature from leaving the sensor FOV, but has the added advantage, albeit indirectly, of angling the sensor in such a way that provides an improved view of the corridor during the disturbance. This is illustrated in Figure 5.8 that shows the simulated view of the feature during the 50% gust for both FS1 and FS2.

![Simulated view of feature for FS1.](image)

![Simulated view of feature for FS2.](image)

**Figure 5.8:** Simulated view of infrastructure during the peak of the 50% gust disturbance in conjunction with the results of Figure 5.7. During this time, the UAV is subjected to a momentary raise in wind speed from the original mean wind conditions of 37 km/h (20 kn) to 55 km/h (30 kn).

The final series of simulations would test the control response in the presence of longer, sustained gusts, acting over a period of 50 s each, altering mean wind by a factor of ±20%, for which mean wind conditions were once again set to 37 km/h (20 kn) acting at 70° relative to north. Figure 5.9a shows the results for both the responses of FS1 and FS2, where the changes in wind conditions are shown in Figure 5.9b. Each of the disturbances is seen to result in steady state **Sensor Track Error**, and although not ideal, would still provide adequate conditions for continued data capture.

This result can be expected however as the controller relies on an estimate of current wind conditions to correctly compensate for the effects of wind, thus any error in that estimate can be expected to directly impact performance. On a positive note however, even under wind estimate errors of ±20%, both controllers are seen to maintain relatively small steady state error, with FS2 providing the
best results, maintaining the feature within 10% of the image centre. In future work an estimate of the wind given current tracking conditions could possibly solve both problems, providing an estimate of wind that can be applied to the tracking controller.

![Image of Sensor Track Error and Atmospheric Disturbances](image)

**Figure 5.9**: Control response of FS1 and FS2 to sustained gusts over 50 s, altering mean wind (37 km/h (20 kn) acting at 70° relative to north) by a factor of ±20%.

A final point worth noting is the overall effect of Turbulence on the control response that was present throughout each of the previous simulations. In general, the effect was slight motion of the sensor LOS, more noticeable during steady state tracking, and seen to increase steadily between the 18 km/h (10 kn) winds of Figures 5.3 and the 28 km/h (15 kn) winds of Figure 5.5, to the point in Figure 5.7, where wind speed has increased to 37 km/h (20 kn), and the result sees features move in a range of 10 pixels (recalling a 2048 × 1536 pixel sensor). This of course can be expected given Turbulence increases as a function of mean wind conditions.
5.5 Summary

This chapter has presented a novel solution to enable the IBVS control design of Chapter 4 to provide simultaneous tracking and data collection through the use of Forward-Slip manoeuvres in the presence of wind, provided an estimate of mean wind conditions is available. The effect of a constant wind disturbance on the original control design was shown to alter the desired line angle from a previously assumed vertical orientation to that of an angle equal to the Wind Correction Angle. Through simulation it was found that the original controller would no longer centre the feature in the presence of a constant wind, which in strong winds could see the feature leave the field of view altogether.

The inclusion of integral control resulted in adequate compensation for light cross winds of 18 km/h (10 kn), although did so at the penalty of overshoot and maintaining a Forward-Slip during steady state tracking. Maintaining such a manoeuvre not only reduces efficiency but results in non-wings level flight that angles sensors away from vertical such that the feature is viewed at an angle. While infrastructure would still appear centred in the FOV, the angled view could have subsequent impact on data processing techniques developed based on a downward facing FOV, e.g. tree crown delineation algorithms that assume an overhead view of the tree crown [25]. At higher cross wind speeds of 28 km/h (15 kn), compensation from the integral controller was found to further degrade, resulting in unwanted motion that could hinder data collection.

The inclusion of a Wind Correction Angle as the desired line angle was shown to improve the response in wind, avoiding overshoot, excessive control surface actuations, and allowing steady state tracking with a wings level attitude. The controller was found to operate well in the presence of Turbulence, with only minor motion of the sensor LOS observed. Adequate suppression of Isolated Gusts up to 50% of mean wind conditions were found, maintaining both the feature and surrounding corridor in the sensor FOV, with the FS2 form of the Forward-Slip control design maintaining the feature within 20% of the image centre. Sustained Gust disturbances were found to result in steady state Sensor Track Error that would remain without an updated estimate of mean wind conditions. Observing such conditions could provide an avenue for future work, estimating wind conditions that would benefit both tracking and other subsystems.
Previous chapters have presented control concepts that enable simultaneous tracking and data collection for a UAV operating a downward-facing imaging sensor over linear infrastructure. This however represents but part of the data collection process, where on a larger scale, infrastructure is “locally linear” and comprises of segments that allow the feature to change direction. While solutions through path planning have been proposed to negotiate such transitions whilst providing favourable position and orientation for downward facing sensors [22, 118], solutions for real-time visual control are limited to that of Egbert and Beard who propose roll and altitude constraints to ensure features remain visible [119]. Controlling the view of the transition from the perspective of a downward facing sensor through visual control is a concept that has yet to be explored.

The following chapter seeks to develop techniques that can extend the straight line following concepts of previous chapters to fulfil this duty, providing simultaneous tracking and data collection through transitions. The chapter begins with the development of a technique that facilitates the transition between segments based on the view of both features in the image plane. This is then applied to both the Skid-to-Turn IBVS controller of Chapter 3 and the Forward-Slip IBVS
controller of Chapter 4, where each controller is assessed on the ability to provide continuous data capture as the angle of transition increases. These tests are then repeated over a range of operating conditions and flight configurations including variations in height, airspeed, wind and the use of separate sensors for tracking and data collection.

### 6.1 Problem Formulation

A practical consideration that has yet to be addressed in previous Chapters 3, 4 and 5, is the ability to transition between linear segments of infrastructure that see the feature change direction. Given the discontinuous nature of transitions, the tracking controller will require decision criteria to effectively manage the process of switching between current and future segments as they enter the sensor FOV. One might anticipate that switching immediately to the future segment as it enters the sensor FOV may provide the best result; affording the controller the longest time possible to alter course for the new segment. Switching in such a manner though may present issues as tracking will change abruptly, potentially inducing unwanted motion as the controller begins tracking the new feature. Ideally the transition would be smooth and continuous, slowly shifting emphasis from the current feature to the new feature as the point of transition moves through the image plane. A solution to this is proposed through a **smoothing feature** that provides a temporary line feature for vision based tracking as the new segment enters the sensor FOV.

Figure 6.1 illustrates the concept of the proposed smoothing feature through a sequence of images as would be captured by a downward facing sensor during the transition shown left. As the UAV approaches the transition, the new segment enters the sensor FOV from the top of the image frame, as illustrated in sample image (1), and will be the first instance the controller sees the transition. Given the short length of the new linear feature and the cluttered environment in which feature extraction takes place, it would be expected that estimates of both Sensor Track Error and Observed Line Angle would be neither accurate nor reliable and would therefore not be used for control until the feature is clearly visible. During this time a more reliable feature would be the point at which the new segment enters the frame. This is approach taken by Holt in [11], where the point of entry
Figure 6.1: Example of the smoothing feature that would be generated during a 25° transition with the UAV following a path shown left. The temporary line feature (blue dashed) is generated between the top and bottom of the image frame, connecting the exit point of the current feature with the entry of the new. Emphasis is shifted between the two segments, as illustrated in the sequence 1 → 2 → 3, allowing a visual controller to switch tracking between the two in a smooth manner.

of a road is used as a virtual target for visual tracking. In this work however the point is used to generate a temporary line feature for tracking.

This is achieved by connecting the exit point of the current feature, as it leaves the lower edge of the image frame, with the entry point of the new, as illustrated by the blue dashed line in the image sequence of Figure 6.1. The line following IBVS controllers of Chapters 3 and 4 can then be used to track the new feature that will in turn see the UAV negotiate the transition. As the turning point of the transition moves through the image frame, the angle of the temporary line feature
will shift towards that of the new segment until finally aligning as the current segment leaves the sensor FOV. Sharper transitions would be compensated in this way with the smoothing feature shifting at a faster rate to present a larger error in angle to the visual tracking controller. By handling the transition in such a manner, feature parameters of Sensor Track Error and Observed Line Angle supplied to the IBVS controller change in a smooth and continuous manner that should limit unwanted motion.

6.2 Test Cases

Testing would begin by assessing the use of the smoothing feature in conjunction with both the Skid-to-Turn (STT) IBVS controller of Chapter 3 and the Forward-Slip (FS) IBVS controller of Chapter 4 and their performance when applied to vision based control for negotiating transitions. The first test case would compare the use of the smoothing feature against two alternate techniques; the first switching tracking features as soon as the new segment enters the FOV and the second switching once the new feature is visible in the top quarter of the image frame. The first technique is only practical in simulation as the new feature would be of inadequate length as it first enters the sensor FOV for estimating line parameters. The second technique delays switching until the feature is a quarter of the way through the image frame where the feature would be of practical length for feature extraction.

Testing would be performed over a 10° transition, utilising the simulation environment as described in Appendix B. Altitude and airspeed would be set at 50 m and 70 km/h respectively for both controllers. The STT IBVS controller would use the same tunings as described in Section 3.3, while the FS IBVS controller would be retuned on account of the original tuning in Chapter 4.2 calculated at an airspeed of 100 km/h. This is achieved by including the new airspeed of 70 km/h in the state space model of the system (Section 4.1.5, Equation 4.29) that is in turn used to generate the optimal state feedback control gain through an LQR solution using the same values of $Q_{2,2}$, $Q_{3,3}$ and $Q_{5,5}$ as in the design of FS2 (4.34) that were 5, 0.5 and 1. The resulting gain matrix is,

$$K_{(70\text{km/h})} = \begin{bmatrix}
0.20 & -1.0 & -0.80 & -1.4 & -0.72 & 0.51 & 5.4 & 0.44 \\
-0.23 & -0.0027 & -0.68 & -4.8 & 0.69 & 4.4 & 0.44 & 1.36
\end{bmatrix} \quad (6.1)$$
The next series of test cases would assess the performance of the two controllers negotiating transitions at increasing angles and the effect of varying operating conditions and flight configuration to improve these limits. It would be expected that beyond these limits an alternate manoeuvre would be performed to enable continued data collection that could include previously proposed techniques of ‘go-around’ or ‘clover-leaf’ style manoeuvres [26, 27]. In addition to ensuring a feature remains visible during tracking, the test cases would also assess the impact on capturing a region of interest either side of the feature. Examples of this include power line corridors, that are routinely inspected for vegetation encroachment, and tracking tasks where only part of the feature is detected for the purpose of tracking, e.g. the centreline of a road where data capture over the width of the road is also of interest.

The first of these test cases, and the second overall, would be performed based on conditions for maximum coverage, where the UAV would be flown at an altitude to capture the full width of the inspection area and minimise data capture outside this region. For an inspection sensor with a horizontal angular FOV of 50°, as used in this simulation, this condition is achieved at an altitude of 50 m. Airspeed would be set at a low cruising speed of 70 km/h (38 kn). The third test case would assess an increase in altitude on the performance of negotiating transitions, raising altitude from 50 m to 100 m. This can be expected to increase the sensor footprint and thus afford the controller greater time in detecting the transition and initiating a change in heading.

To provide a fair comparison with results at the original altitude, both the STT controller of Chapter 3 and FS controller of Chapter 4 were retuned for the new operating height. This varies from test cases of previous chapters where variations in altitude were tested with gains held constant, thus testing the controllers robustness to varying operating conditions. In these tests the change in altitude is intended and therefore would see the controller tuned for the given flight condition. In the case of the STT controller, Sensor Track Error as measured in the image plane is scaled by a factor of 2 to compensate for the scaling introduced by the change in altitude and avoids adjusting the gains previously set in Section 3.3.

Recalculating gains of the FS controller is far simpler, where the new height can be included in the state space model of the system (Section 4.1.5, Equation...
Transitions

4.29) and used to update optimal state feedback gains by repeating the LQR solution. Weighting matrix $Q$ is again selected from the original design of FS2 (4.34), that with the addition of the height results in a new gain matrix,

$$K_{100 m} = \begin{bmatrix}
0.21 & -1.0 & -0.76 & -1.3 & -0.74 & 0.45 & 5.4 & 0.38 \\
-0.19 & 0.021 & -0.86 & -4.5 & 0.68 & 3.0 & 0.38 & 1.6 
\end{bmatrix} \quad (6.2)$$

The fourth test case would assess negotiating transitions at higher airspeeds, increasing airspeed from 70 km/h (38 kn) to 100 km/h (54 kn), while remaining at an altitude of 100 m. Selection of airspeed for a given inspection task can be expected to vary on account of a number of factors including improved efficiency, compensation for additional payload and controlling overlap in data capture. It is therefore of interest to assess the effect of varying airspeed on negotiating transitions. Control gains for the STT IBVS controller would remain constant given that previous results in Chapter 3.4 found an increase in airspeed improved performance. State feedback gains were recalculated for the FS controller, incorporating the new airspeed in the state space model of the system (Section 4.1.5, Equation 4.29) that is in turn used to generate the optimal state feedback control gains through an LQR solution. The weighting matrix $Q$ was again adopted from control design FS2 (4.34), resulting in the gain matrix,

$$K_{100 km/h} = \begin{bmatrix}
0.23 & -1.0 & -0.72 & -0.13 & -0.95 & -0.43 & 8.0 & 0.48 \\
-0.13 & 0.036 & -0.80 & -2.4 & 0.32 & 2.4 & 0.48 & 2.5 
\end{bmatrix} \quad (6.3)$$

The fifth test case would assess transitions with the use of separate sensors for data collection and tracking. One benefit of controlling sensor FOV via control of aircraft motion is the follow on effect this has on other sensors with equal alignment, with these gaining the same benefits as the tracking sensor. This can prove particularly useful in separating the tasks of tracking and data collection, where tracking is likely to benefit from a larger sensor footprint, increasing the time to detect approaching transitions, while data collection benefits from a narrow FOV allowing focus on the inspection region. This concept forms the fifth test case, where the focal length of the sensor collecting data is doubled to provide a horizontal FOV of 27°, while the tracking sensor would continue to operate a sensor with a horizontal FOV of 51° as in previous test cases.

The final series of tests would assess the performance of the controllers under
moderate cross wind conditions of 28 km/h (15 kn) with respect to the feature. The sixth test case would assess control performance in unknown wind conditions, requiring both controllers to utilise integral control to account for the effects introduced by wind during tracking. The seventh and final test case would then assess the performance of the FS controller of Chapter 5 where an estimate of mean wind conditions are available for control. The STT controller of Chapter 3 is not included in this test case as the controller was developed to operate independent of an autopilot and other onboard systems, and would therefore not have access to this information.

6.3 Results

Figure 6.2 shows the results of using the proposed smoothing feature in conjunction with the STT controller of Chapter 3 to negotiate a $10^\circ$ transition. Also included is a comparison with switching immediately (immediate) and switching once the feature is a quarter of the way through the image frame (switching @ 1/4 Frame). In the former, the STT controller begins to track the new segment as soon as it enters the frame, while the latter represents a more realistic scenario where the new feature is only tracked once large enough for feature extraction to reliably estimate line parameters. It should be noted at this point that Sensor Track Error ($T_e$), as shown in Figure 6.2 and subsequent figures in this chapter, provides a direct measure of $T_e$ with respect to the actual feature as seen in the image frame. This differs from Sensor Track Error that is supplied to the controller during the transition. The reason this value is not shown is that during the transition the controller is tracking a temporary line feature generated by the smoothing feature. This provides no real measure of where the feature actually lies in the image plane, hence Sensor Track Error that is shown is always relative to the feature in the image plane.

As $T_e$ is measured with respect to one line feature, the use of the metric when two features enter the frame as a transition becomes ambiguous. To overcome this issue and present a useful metric for assessing performance, the value is switched as the two features pass the image centre, where $T_e$ is measured relative to current line until the point of transition passes halfway through the image and is then measured with respect to the new feature.
Switching immediately is seen to be the least effective solution for negotiating transitions when using the STT controller, where the UAV overshoots the corner by approximately $T_e = 30\%$ before reacquiring the line in the image centre after 8 s. Switching later as is the case in switching @ 1/4 frame shows better performance, while the smoothing feature offers the best result limiting overshoot to $T_e = 22\%$. The poor performance of switching immediately can be explained by considering that when the controller switches to the new segment, the new feature presents $T_e$ that will minimised by continuing to fly ahead, thus leaving the controller to turn later. In the case of the smoothing feature the controller is provided a temporary line feature that has both $T_e$ and a line angle that encourages the controller to turn earlier.

Figure 6.3 shows the response of the FS controller of Chapter 4 when used in conjunction with the smoothing feature to negotiate the transition. Unlike the STT controller, the FS controller is actually seen to benefit from immediate switching. This can be explained by the FS controller utilising bank to recentre the feature that actually works in favour of negotiating the transition. Previously in Chapters 4 and 5, the response of the FS controller recentring a feature from initial cross track error, saw the controller command the aircraft into a forward-slip manoeuvre. This allowed the UAV to bank sensors towards the feature,

\footnote{This should not be confused with 30\% overshoot, rather $T_e$ is measured as a percentage of image width, i.e. 100\% $T_e$ reflects a line that is at the edge of frame.}
6.3 Results

Figure 6.3: Response of the FS IBVS controller to a $10^\circ$ transition using the smoothing feature. Comparison is made with immediate switching and switching as the feature passes the top quarter of the image frame.

while counteracting the turn that would see the UAV fly away from the feature otherwise. In the case of the transition though, the controller no longer requires forward-slip in the same manner, as the bank angle induced to centre features in the sensor FOV works with the UAV negotiating the transition. Thus detecting a transition earlier will allow the controller to access larger angles of bank that can assist the controller negotiating sharper angles of transition. In other words, as would be expected of a position-based controller, the earlier the controller can detect and initiate a turn, the larger the angle of transition the UAV can be expected to negotiate.

Given immediate switching is not practical, based on the feature being of inadequate length to detect line parameters $T_e$ and line angle, the next best option would be to switch once a sufficient length of the feature is within the image frame. This is the solution presented by switching @ 1/4 frame, where the FS controller is supplied the new line feature once the point of transition passes the top quarter of the image frame. This does result in reduced performance compared to immediate switching that only incurred overshoot of approximately $T_e = 13\%$ that increases to $16\%$ for switching @ 1/4 frame, although improves over the smoothing feature that incurs overshoot of $T_e = 19\%$. Based on these results, the FS controller from this point forward would use switching @ 1/4 frame, while the STT controller would using the smoothing feature.

The next series of tests would assess the performance of both controllers over increasing angles of transition. Figure 6.4 shows the results for the second test
case, with the UAV operating at an airspeed and altitude of 70 km/h and 50 m respectively; a flight condition that allows for maximum coverage of the inspection area, i.e. allows the inspection region to fill the sensor FOV. The FS controller is seen to have a clear advantage over the STT controller, reducing overshoot by 47% and 37% for 10° and 15° transitions respectively, while also reducing the time taken to recentre the feature in both cases by a factor of 65%. The improved performance of the FS controller does however come at a cost, with the controller generating greater angles of sideslip compared to those induced by the STT controller. Considering both controllers operating within the same limits of sideslip, in this case ±15°, the STT controller is able to complete a 20° transition before reaching limits of sideslip, while the FS controller is reaching the same limits during a 15° transition.

Figure 6.5 provides an alternate representation of the results from Figure 6.4 with sensor coverage illustrated from an overhead perspective of the simulated inspection area. The physical path flown by the UAV is shown in blue, while the projected line-of-sight (LOS) of the sensor at ground level is shown in red. Sensor coverage is highlighted in three different shadings, with white representing coverage within the inspection region that is 20 m either side of the feature, light red where sensor coverage falls outside this region, and yellow to indicate areas of the inspection region missing coverage. Both controllers are seen to provide full coverage of the inspection corridor, where the concept of maximum
**coverage** is highlighted by the minimal amount of data capture that falls outside the inspection area, as indicated in light red.

As a result of maintaining wings-level flight throughout tracking, the STT controller is seen in Figure 6.5a to fly a path over ground that aligns with the projected LOS of the sensor. This differs from the FS controller that utilises rotations to perform both bank-to-turn and forward-slip manoeuvres at different stages to assist tracking. This is highlighted in Figure 6.5b where the controller is first seen to initiate a small bank-to-turn manoeuvre as it passes over the power pole. This angles the sensor LOS to the right of the flight path and subsequently away from the feature, although allows the UAV to alter heading that is necessarily to track the new segment. A short while later the controller banks the aircraft in the opposite direction, allowing the sensor to point towards the feature that is indicated by sensor LOS realigning with the feature after the transition. This requires the UAV to perform a forward-slip manoeuvre that is indicated by the sensor LOS being angled to the left of the flight path, while the flight path also moves to the left over time, a combination that is made possible through a forward-slip manoeuvre.

![STT Controller](image1.png)  
(a) STT controller response to transition.  

![FS Controller](image2.png)  
(b) FS controller response to transition.

**Figure 6.5**: Overhead view of a simulated inspection region during the 15° transitions associated with the results from Figure 6.4. Inspection region surrounding the feature is ±20 m. (Altitude - 50 m, Airspeed - 70 km/h)

Figure 6.5 also highlights an issue that leads to both controllers incurring overshoot during the transition. Observing the point along the flight path where the UAV begins to alter heading, it is seen to occur very close to, if not after, the
UAV reaches the power pole that forms the transition. This can be attributed to the distance at which the sensor is detecting the transition, where sensor footprint at this height is approximately $42 \times 34$ m that results in the sensor detecting features forward of the aircraft at approximately 17 m. Considering the UAV is flying at 19.4 m/s, overshoot would appear inevitable. The situation can though be expected to improve as the projected footprint of the sensor increases, that is achieved through both increases in altitude and the use of a wider angle lenses.

Figure 6.6 shows the results of the third test case, with the UAV flying at an increased altitude of 100 m to effectively double the length and width of the sensor footprint from the previous test case.

![Sensor Track Error](image)

**Figure 6.6**: Transition response of both the STT IBVS controller and FS IBVS controller when presented with transitions of increasing angles operating at an increased altitude of 100 m. (Altitude - 100 m, Airspeed - 70 km/h)

The increased footprint is seen to considerably improve the response of the STT IBVS controller, with the controller now negotiating transitions of 35° compared to 20° found previously when operating within the same limits of $\pm 15^\circ$ sideslip. Overshoot is considerably reduced and not only on account of Sensor Track Error decreasing in scale, which given the increase from 50 m to 100 m altitude is a factor of approximately 2. Comparing the response of the 20° transition in Figure 6.4 at 50 m altitude to that of the 20° transition at 100 m altitude in Figure 6.6, the STT controller shows a 69% reduction in overshoot. The response of the FS IBVS controller to transitions is also seen to improve at increased altitude. The controller is now able to achieve transitions of 25° compared to 15°
previously while operating with the same limits of sideslip. While overshoot is seen to improve with increased altitude, it should be noted that the reduction is as a result of the scaling of Sensor Track Error with respect to height. An example of this is seen between a comparison of responses at 50 m and 100 m for the 15° transition that reveals a 50% decrease in overshoot that corresponds to the same overshoot scaled for altitude.

Previously at the lower altitude of 50 m, the FS controller was found to reduce overshoot compared to the STT controller for the same angle of transition. This is no longer the case at the increased altitude, where a comparison of both the 20° and 25° transitions reveal that both controllers reach similar levels of overshoot. If this trend continues, the STT controller can be expected to offer reduced overshoot in comparison to the FS controller above certain altitudes. The time taken to recentre the feature after the transition is seen to be invariant to both the angle of the transition and change in altitude for both controllers. This is reflected with STT and FS control responses recentering the feature consistently in 17 s and 6 s respectively, reflecting the same 65% reduction previously found for the FS controller at the lower altitude of 50 m. These results present an interesting compromise between the controllers, with the FS controller offering faster times to recentre the feature, while the STT controller offers the ability to negotiate larger transitions within the same limits of sideslip and the potential to reduce overshoot at increased altitudes.

Figure 6.7 shows data capture as would be achieved over inspection regions associated with the 25° transition performed by the FS controller and the 35° transition performed by the STT controller from the results of Figure 6.6. Both controllers are seen to capture the full width of the inspection region, where the increase in altitude is seen to expand the sensor footprint in both height and width by approximately twice that of the original. The point at which the two controllers begin to alter heading is now seen to be earlier and can be attributed to the larger footprint that allows earlier detection of upcoming transitions. The FS controller is again seen to initiate a change of heading through a small bank-to-turn manoeuvre, that is evident from the sensor LOS being angled to the right of the flight path during the beginning of the transition.

Figure 6.8a shows the results of the fourth test case for an increase in airspeed on the response of the STT controller negotiating a 35° transition. It should be noted that the control design itself does not allow for direct compensation
Figure 6.7: Coverage achieved by inspection sensor for both STT and FS controllers (light red) with the UAV operating at an increased altitude of 100 m in association with the results of Figure 6.6. Inspection region (white) extends ±20 m either side of the feature.

for changes in airspeed and is thus left with original gains of the previous test case. The increase in airspeed is seen to improve the controllers response to the transition, reducing both the time to recentre and overshoot by factors of 35% and 17% respectively. Figure 6.8b shows the response of the FS controller to the 25° transition at the increased airspeed of 100 km/h. This control design is relatively easy to adjust for variations in airspeed and thus gains are re-calculated to match the operating conditions. The result would appear to be very similar at first, although it should be considered that the result is shown with respect to time, while the UAV is covering greater distances at the increased airspeed. Thus while the increased airspeed would appear to provide a slight improvement in response, reducing overshoot by approximately 22%, it actually increases the distance along the feature at which point the feature is recentred.

While increasing the altitude, and thus sensor footprint, had a positive effect on negotiating transitions, the impact on data collection is an increase in the area captured outside the inspection region. This reduces the percentage of image frame or sensor FOV that contains useful information, that would ideally contain only the region of interest. The benefit of controlling the FOV of one sensor via appropriate control of aircraft motion during tracking is the effect is passed onto any other sensor fixed to the aircraft with equal alignment. This proves particularly useful in separating tracking and inspection that have opposing requirements; with tracking benefiting from a larger sensor footprint, increasing the
6.3 Results

Figure 6.8: Response of the controllers negotiating transitions at higher airspeeds of 100 km/h (54 kn). (Altitude - 100 m, Airspeed - 100 km/h)

(a) STT controller response to 35° transition. (b) FS controller response to 25° transition.

time to detect approaching transitions, and inspection benefiting from a smaller FOV to maximize coverage of the inspection region.

This is the concept adopted in the fifth test case, where the focal length of the sensor collecting data is doubled to provide a horizontal FOV of 27°, while the tracking sensor would continue to use a sensor with a horizontal FOV of 51° as in previous test cases. Given only the inspection sensor FOV has been altered, results over varying transition angles can be expected to be a scaled version of those presented in Figure 6.6. The main advantage is seen in coverage, as illustrated in Figure 6.9 that shows a comparison of the new concept (c) versus the result of flying a single sensor at low altitude (a), and operating the same sensor at higher altitude (b).

Although the point at which the aircraft begins to alter course is only moments earlier, the effect on tracking is quite noticeable, not only allowing the corridor to be fully captured but also maintaining the feature closer to the image centre. Comparison between the tracking sensor coverage (b) and inspection sensor (c) highlights the shared line-of-sight that benefits aligned sensors. Of course any misalignment between sensors could equally result in poor data collection conditions, requiring care during setup and calibration. Another factor to consider, although not as obvious from the results of Figure 6.9, is increased motion observed by the inspection sensor that now observes a scaled version of motion seen by the tracking controller. Suppressing motion from the perspective of the tracking sensor then becomes very important, particularly as the difference in focal lengths between sensors increases.
Figure 6.9: Example illustrating the benefits of using separate sensors with one providing feature parameters for control and tracking, and a second for data collection. Example compares one sensor, flying low (a) and high (b) to the benefits of using two separate sensors (c).

The sixth test case would assess the performance of the controllers under moderate wind conditions of 28 km/h (15 kn) acting perpendicular to the segment prior to the transition. Initially control performance is assessed in unknown wind conditions, where results are shown in Figures 6.10 and 6.11 for STT and FS controllers respectively. In both instances, controllers are seen to adequately compensate for the effects of wind during the transition. While wind may be expected to hinder the performance of tracking when acting against the corner, as is the case of Figures 6.10a and 6.11a, response is actually seen to be satisfactory,
and can be attributed to the aircraft already compensating for wind that sees heading already angled towards the transition. The opposite is also experienced however when wind is acting in favor of the transition that may be expected to assist the turn, however given the UAV now compensates prior to the transition by altering heading away from the transition, any benefit from the wind is actually required to account for the addition change in heading.

Figure 6.10: Response to 30° transition for STT controller in the presence of an unknown, 28 km/h (15 kn), wind acting from both west-to-east and east-to-west.

The response of the STT controller, while allowing the feature to vary from the image centre by small amounts, is seen avoid any unwanted motion throughout the tracking process. The FS controller on the other hand provides more accurate centring of the feature before and after the transition, although is seen to incur unwanted motion in the west-to-east wind case of Figure 6.11a, with small oscillation of the sensor during the transition, although provides a smooth transition during the east-to-west case of Figure 6.11b.

An interesting point to note is the variation between sensor LOS and flight path as a result of tracking in the presence of wind. In the case of the STT
controller, the variation can be attributed to steady state pitch that sees the sensor angled slightly forward of vertical. When operating in no wind the effect is a slightly forward view of the feature, however in wind, where heading no longer aligns with course, the sensor angles away from the flight path, an effect compensated by the controller. Variation in sensor LOS and flight path of the FS controller on the other hand is seen to be much larger and is rather the result of the controller utilising forward-slip to compensate for the effects of wind.

The final series of tests would assess transition response when an estimate of mean wind conditions where available. This would only be assumed available for the FS controller, as the STT controller was developed to operate independent of an autopilot and other onboard systems, and has already shown adequate compensation through the integral method of wind compensation. As would be expected, the response of the FS controller with the addition of the Wind Correction Angle improves the controllers response in the presence of wind, avoiding

\[ \text{(a) Transition response in west-to-east wind.} \]

\[ \text{(b) Transition response in east-to-west wind.} \]

\[ \text{Figure 6.11: Control response of the FS IBVS controller negotiating a 30° transition in the presence of an unknown 28 km/h (15 kn) wind, acting from both west-to-east (a) and east-to-west (b). Controller is seen to maintain forward slip during tracking as evident from the projection of the line-of-sight (red) away from the flight path (blue).} \]
the oscillation observed previously during the west-to-east wind. In terms of coverage however, the response is now seen to miss a small section of the corridor at the very inside of the transition, although was previously captured when utilising compensation techniques in the case of Figure 6.11.

(a) Transition response in west-to-east wind. (b) Transition response in east-to-west wind.

Figure 6.12: Response of the FS IBVS controller negotiating a 30° transition in the presence of a known 28 km/h (15 kn) wind.

The can be explained by observing the means by which the controller previously compensated for wind. In the case of unknown wind, the controller flew at a constant forward slip that sees the sensor angled away from vertical and towards the feature, as was evident between the aircraft flight path and projected line-of-sight. In either wind scenario this can be seen to assist data collection, either flying the aircraft closer to the inside of the corner as in the case of Figure 6.11a, or angling the sensor towards the inside of the corner as in the case in Figure 6.11b. So while data is collected over these areas, it would be at an angle, while the response of Figure 6.12 would be more desired given the downward facing orientation held for the majority of tracking. In any event, increasing sensor footprint via increased altitude or the use of a wider lens would address these issues, and is rather discussed here to highlight practical considerations.
6.4 Summary

This chapter has addressed vision based control for negotiating transitions of locally linear infrastructure through the development of switching criteria that enable the extension of previous developments to fulfil this task. A solution is presented in the form of a smoothing feature that provides a temporary line feature for tracking while both the current and future line segments are visible in the image frame. Both the STT IBVS and FS IBVS controllers of Chapters 3 and 4 respectively, were tested over a range of transition angles. The main limiting factor was found to be the distance at which the controller was able to detect and thus initiate a transition given the relative restriction of detecting the transition from a downward facing sensor. Increasing sensor footprint was found to significantly improve performance that is readily achieved through an increase in altitude, or through the use of a wider lens.

The FS controller was found to provide the best performance through transitions, incurring minimal overshoot and faster times to recentre the feature after completing the transition. In this particular example, flying at 100 m operating a sensor with a horizontal FOV of 50°, the FS controller was able to successfully negotiate transitions up to and including 25°. On account of inducing less sideslip, the STT controller was able to negotiate transitions up to 35° in the same test. The downside of this being an increase in overshoot and a longer time to recentre the feature compared to the FS controller. Control of aircraft motion to assist a body-fixed tracking sensor is shown to aid additional sensors with equal alignment, utilised here to separate tracking and inspection tasks between independent sensors suited to each task.

It should be noted that the limits discussed here represent one particular flight configuration, where in practice, limits will vary dependent on the sensor footprint and the ability to alter heading through STT and FS. These factors can however be directly anticipated and the height and selection of sensors made to accommodate the inspection task given general knowledge of transitions that can be expected and the desired width of coverage surrounding the infrastructure. In general though, the FS controller would be more suited to infrastructure with shallow bends (e.g. pipelines), while the STT controller would provide improved tracking for those with larger transitions (e.g. power lines).
Conclusion

As UAV technology makes its way into the civilian sector, one of the principal applications likely to be adopted by industry is the inspection of infrastructure. A crucial step in enabling automated inspection services is the automated collection of data, a task well suited to UAVs in general, in particular fixed-wing platforms for those inspection tasks spanning large areas. Infrastructure in rural and remote areas represent some of the largest and most widespread networks requiring inspection and given their remote and isolated nature are likely to attract the focus for initial trials. The use of fixed-wings UAVs in these applications would appear inevitable, thus the development of techniques that can address issues that may hinder or impede their use in these applications not only has significance in research but also relevance in industry.

In the early stages of this research a series of flight trials were conducted utilising UAV systems to collect data for the purpose of inspection over power lines in rural Queensland, Australia. During these trials two key issues were identified. The first concerned dealing with unreliable location information of infrastructure, where inaccuracy in such information was found to lead to poor data collection when utilised in path planning for UAVs executing data collection.
The second issue concerned data collection itself, as the means by which fixed-wing UAVs manoeuvre to alter heading was found to directly influence data collection. In particular, those sensors in a downward facing orientation were subjected to rotations that lead to rapid movement of features that had the potential to leave the FOV altogether during large manoeuvres.

The development of new techniques to solve these issues formed the focus of this research, in particular, those that would allow direct control of the task from the perspective of an inspection sensor. An extensive review of literature revealed that current developments in the fields of automated inspection, UAV path planning and following, and vision based control, either:

- provided a solution through path planning assuming accurate knowledge of infrastructure location;

- compensated for vehicular motion and feature offset through a gimballed system;

- utilised position based visual servoing techniques to enable real-time tracking, however posed the task in terms of flying directly over a feature, as opposed to providing an optimal view; or

- utilised image based visual servoing techniques that allows control of a desired view, however are addressed through forward facing sensor that avoids much of the unwanted motion that affects downward facing sensors.

The use of gimballed mounts is a common solution for enabling fixed-wing UAVs to maintain point targets in the FOV of sensors given they can be controlled independent of vehicle orientation. The systems do however have their limitations and are not necessarily an all in one solution to the problem. For inspection tasks that require the UAV to operate within close proximity of the feature, the compensation a gimbal can provide is not necessarily ideal, as the act of angling sensors away from vertical alters the perspective at which the feature is viewed. This can have subsequent impact on automated data processing algorithms that rely on consistent viewing angle for correct operation, e.g. tree crown delineation that requires an overhead view of the tree crown. There are also instances where the size, weight and added complexity of integrating inspection sensors onto a gimballed mount is neither practical nor feasible, in particular,
with small platforms. Thus controlling vehicular motion in such a way to assist data collection can equally improve systems with sensors both body-fixed and gimbaled.

The use of vision in the control and guidance of UAVs would at first appear to address many of these issues, providing real-time tracking while, by principle of design, requiring the feature to remain visible. Unfortunately many of the solutions that provide visual tracking are posed through Position Based Visual Servoing techniques that, while providing real-time tracking, pose the control task in the 3D task space that remains focused on flying directly over a feature; providing limited control over a desired view, with the risk of features leaving the sensor FOV. Of those visual control solutions that do provide direct control over a desired view of features during tracking, solutions are limited to forward facing sensors that include Image Based Visual Servo control for fixed-wing UAV landing.

From this review, a gap in knowledge was identified that presented a potential avenue for novel contributions to be made in the guidance and control of UAVs. This formed the overall goal of the research, seeking a solution that provided:

- real-time tracking,
- direct control of the inspection task from the perspective of a downward facing sensor,
- means to obtain a desired view of a feature during tracking, and
- the ability to minimise unwanted motion that could otherwise corrupt data.

In light of this, the research posed three research questions that were considered open research topics, that once addressed, would contribute to gaps in knowledge and provide enabling technology towards the overall objectives. The first posed:

*Can a fixed-wing UAV be controlled in such a way to provide tracking of locally linear infrastructure whilst enabling data collection from a downward facing sensor?*

Through the developments of Chapters 3 and 4 two distinct manoeuvres emerged as providing the necessary control to track a linear feature whilst providing adequate conditions for data collection from a downward facing sensor. The
first manoeuvre that was investigated was the Wings-Level Skid-to-Turn that provides means to alter heading whilst maintaining a wings-level orientation that avoids much of the unwanted motion coupled to downward facing sensor that results from Bank-to-Turn manoeuvres. The second manoeuvre was discovered as part of an optimal control solution developed in Chapter 4 and was identified as a special use of sideslip known in general aviation as the Forward-Slip. Although generally reserved for landing on short runways where increased drag is not available through flaps, the manoeuvre was found to have interesting properties that could be exploited for the purpose of inspection. The manoeuvre not only avoids unwanted motion, but provides means to centre features in one smooth motion, utilising body rotations to first angle downward facing sensors toward the intended target, then slowly altering course through sideslip to bring the vehicle over the feature allowing the aircraft to return to wings level attitude.

The second research question concerned the use of these manoeuvres to optimize data collection based on real-time detection of features from a downward facing sensor and posed:

*Can visual information extracted from a downward facing imaging sensor be utilised to control a fixed-wing UAV in capturing a desired view of infrastructure?*

Image Based Visual Servoing was identified as a possible solution to this problem and one that had not been investigated in this context. In the development of Chapter 3, it was shown that control through vision could maintain a desired view of features with visual error derived from the current view of features that could be minimised through STT manoeuvres to provide a tracking solution that could centre features while avoiding unwanted motion. In the development of Chapter 4, the dynamics of the fixed-wing platform were fully utilised through Image Based Visual Servoing techniques to control the desired view of features with improved performance whilst reducing control effort over STT manoeuvres, while Chapter 5 extended this to provide a desired view of features whilst tracking in the presence of wind.

The third and final question then sought to address negotiating the transition between segments of locally linear infrastructure that present discrete changes in feature course. The controller would not only be required to switch between segments during this period, but also ensure conditions were provided for continued
data collection, leading to:

*Can techniques be developed from visual cues alone that enable the UAV to successfully negotiate the transition between segments while ensuring continued data capture?*

Through the developments of Chapter 6 it was shown that the linear tracking techniques developed in earlier chapters could satisfy the task of negotiating transitions with the addition of switching criteria. In the case of the STT IBVS controller of Chapter 3, this was found to be enabled through the introduction of a smoothing feature that created a temporary line feature for tracking between the entry of the new segment and the exit of the previous. The FS controller of Chapter 4 on the other hand was found to benefit from switching between a current and new line segment as soon as the new line feature entered the image plane, as this allowed the controller to utilise bank-to-turn manoeuvres in negotiating the transition. The maximum angle of transition was found to be directly influenced by the size of the projected sensor footprint as this determines the physical distance at which the transition is detected by the controller. This can be directly influenced by increasing altitude and widening sensor FOV, and thus used to accommodate for larger angles of transition.

In answering these questions this research has made a number of novel contributions that provide enabling technology for improved data collection from fixed-wing UAVs. A first contribution is made through a novel Skid-to-Turn (STT) Image Based Visual Servo (IBVS) controller, that introduces the concept of tracking infrastructure from the perspective of a downward facing sensor, allowing control to be posed directly in terms of data collection. This is achieved using principles of IBVS to command Skid-to-Turn manoeuvres to obtain a Desired Line Angle that is generated as a function of current Sensor Track Error measured in the image plane. The design allows for minimal integration with an existing autopilot system, commanding rudder through vision alone, where a pre-existing autopilot would be commanded to maintain altitude, airspeed and wings-level altitude. The proposed controller is shown to outperform Position Based Visual Servoing (PBVS) utilising Bank-to-Turn (BTT) that is common in literature, reducing feature motion as detected by the inspection sensor by an average of 80%, over a range of operating conditions including variations in altitude, airspeed and wind.
A second contribution is made in the form of a Forward-Slip (FS) Image Based Visual Servo (IBVS) controller, that utilises the full lateral dynamics of the platform by augmenting the interaction matrix of a line feature with dynamic model of the aircraft. The design linearises the interaction matrix of the extracted feature about a desired operating point such that the kinematics of the feature’s motion in the image plane can be augmented with the aircraft lateral dynamic equations of motion. From this, a controller is then developed based on full state feedback through an LQR control design.

This differs from other line following IBVS solutions where the interaction matrix is approximated about the desired pose and selected as the output matrix of the system, from which it can then be inverted for direct control over feature error [185, 189]. The optimal solution of the LQR controller is found to use forward-slip manoeuvres in place of skid-to-turn, that improves performance by 67% while using equivalent levels of control effort to STT, and reducing maximum control surface deflections by over 50%.

A third contribution is made in the form of a vision based control solution for improving data collection when the controller has access to estimates of mean wind conditions. The development introduces a model of mean wind to the interaction matrix of the extracted feature and through linearisation is shown to satisfy steady state tracking with the introduction of the Wind Correction Angle. The design is shown to improve performance over pre-existing integral based techniques that compensate for steady state Sensor Track Error introduced by wind. The solution is not only shown to handle mean wind conditions but also adequately suppress the effects of turbulence and gust disturbances that are likely in an outdoor operating environment.

A fourth contribution is made through the development of techniques to provide vision based control through the transition of locally linear segments. A solution is presented in the form of a smoothing feature that generates a temporary line feature for tracking as two segments enter the sensor FOV. The ability to successfully negotiate a transition is shown to be directly linked to the size of the projected sensor footprint, whereby a larger footprint allows for the earlier detection of new segments and thus provides greater distance over which to alter heading. Given the correlation between operating altitude, sensor angular FOV and projected sensor footprint, it is therefore possible to alter a mission to accommodate for any anticipated transitions along the length of infrastructure.
7.1 Future Work

The overall contribution of this research has seen the generation of new knowledge in the area of guidance and control for fixed-wings UAVs, specifically in the management of data collection from downward facing, body-fixed sensors. In addressing the research objectives and the development of novel solutions to key issues, this research has made an original and significant contribution to knowledge in the field of automation that can provide real world benefits in the future.

7.1 Future Work

The outcomes of this research open a number of potential avenues for future work, these include:

- The implementation of the developed controllers onboard a UAV system for testing in a real world environment. Given the number of supporting subsystems required to test the developed controllers (i.e. UAV Platform, Autopilot, Ground Station and Onboard Image Processing) it was unfortunately not possible at the time of this research to perform these experiments.
  - This could be supplemented with the development of techniques as presented in Appendix B.2 onboard the UAV to provide synthetic feature generation for hardware in the loop testing. This not only removes the need for real-time feature extraction, but also removes the constraint on flying near infrastructure during testing.

- The inclusion of feed-forward control in the STT IBVS control development of Chapter 3. While the current development demonstrates the ability to utilise an existing autopilot to compensate for roll induced through sideslip, a possible extension could improve performance by providing this information to the autopilot to initiate ailerons prior to roll being induced.

- The extension of techniques to provide tracking for a Corridor. This could utilise the same techniques developed for the single line feature, modelling extents of the corridor as two line features, allowing direct control of data collection over an inspection region, e.g. power lines, roads and rivers.

- Redevelopment of the techniques in Chapter 4 to provide a tracking solution for a UAV inspecting a feature at an Angle, i.e. allow a UAV equipped with
a side mounted camera to track a feature off to one side. This would see the interaction matrix couple lateral and longitudinal dynamics that could lead to a number of interesting developments.

- The extension of control to non-linear techniques that could possibly address issues including steady state tracking in wind that at present require an estimate of wind to correct, however may be solved using gain scheduling or similar techniques to extend the operating point.

- The development of a framework to combine the advantages of the two IBVS controllers, utilising the benefits of the FS controller in providing accurate tracking over linear segments and shallow transitions, while utilising STT over larger transitions. This could also incorporate alternate manoeuvres to negotiate transitions that exceed practical limits of the visual controllers.

- The development of an algorithm to calculate transition limits based on the findings of Chapter 6. This could incorporate height and angular FOV, with airspeed and turning performance of the UAV, allowing transition limits to be determined; likewise, for a given transition angle, calculate the required flight parameters to negotiate the transition.

- This could open way for the development of higher level decision making processes, i.e. weighing the options of altering height, airspeed, camera focal length and manoeuvre selection for each transition and the subsequent impact on fuel use, data capture etc.
Mathematical Background

The following section provides an overview of background principles used in the development of this research. Appendix A.1 defines the reference frames used throughout the research. Appendix A.2 details the development of the 6 Degree of Freedom model derived from Newton’s second law of motion, followed by the process of linearising these equations about a trim condition and creating a linearised model for the Aerosonde® UAV. Appendix A.3 defines the camera model used in this research, followed by the derivation of the Interaction Matrix for a point feature in Appendix A.4 that forms the foundation for other developments.

A.1 Coordinate Frames

The following section describes the coordinate frames used throughout the thesis and defines the standard notation. In general, coordinate frames can be separated into those that provide a reference with respect to Earth, e.g. Earth Centred Earth Fixed (ECEF), and those that provide a reference relative to the airframe, e.g. Generalised Body Axes.
A.1.1 Body-Fixed Frames

A total of three body-fixed coordinate frames are used throughout this research and are shown in Figure A.1. The Generalised Body Axes, or unless otherwise stated, body-fixed axes, form a right hand orthogonal axis system centred at \( o_b \) that coincides with the airframe centre of gravity (c.g.), where axis \( o_x b \) is defined as running through the aircraft nose, \( o_y b \) through the right wing and \( o_z b \) directed below.

The second frame is generally adopted when considering aerodynamic forces and rotates the body-fixed axes by the angle of attack angle \( \alpha \) to produce Wind or Stability axes that align with the free stream velocity vector \( V_0 \), shown here as \( (x_w, y_w, z_w) \). The final set of axes defines the camera frame and can be both translated and rotated from the body-fixed axes to give \( (x_c, y_c, z_c) \), where the reader is referred to Section B.2.1 for transforms between these frames.

![Figure A.1: Body-fixed coordinate frames. Note: Camera position is for illustrative purposes only, throughout the control developments it is assumed that the camera is close to the aircraft c.g.](image)

A.1.2 Inertial Reference Frames

Relating motion of the aircraft with respect to ground based features requires a number of reference frames, for which the three used throughout this work are illustrated in Figure A.2. These include the Earth Centred Earth Fixed reference frame, that is assumed inertial given the short term motion considered
A.2 Aircraft Model

and is defined at the Earth centre, with \( oz_e \) aligned towards north along the Earth’s axis of rotation, \( ox_e \) aligned toward 0° longitude, and \( oy_e \) forming a right hand orthogonal set of axes. The second coordinate frame commonly used to locate objects on the Earth’s surface is Geodetic Latitude, Longitude and Altitude (LLA) and defines a point \( P \) at \((\phi, \lambda, h)\). Finally, local motion can be described through a local vertical coordinate system, in this instance selected as North, East, Down (NED), although also commonly defined as East, North, Up (ENU). As the name suggests, the frame is orientated with \( ox_o \) towards north, \( oy_o \) to the east and \( oz_o \) down, where transformation between these frames is detailed in Section B.2.1.

![Diagram of Earth fixed coordinate frames]

Figure A.2: Earth fixed coordinate frames, LLA \((\phi, \lambda, h)\), ECEF \((x_e, y_e, z_e)\), and NED \((x_o, y_o, z_o)\) used in the generation of simulated imagery.

A.2 Aircraft Model

The derivation of the dynamic equations of motion for an aircraft are covered in great detail by many authors, including M.V. Cook [210], D. McLean [209], J. Roskam [216] & R.C. Nelson [208], with the original derivation generally attributed to founding work of Bryan (1911) and Lanchester (1908) [224,225]. Classically the derivation begins with the application of Newton’s second law of motion
to the six degrees of freedom, simply stated for Linear Motion as,

\[ \text{disturbing force} = \text{mass} \times \text{acceleration} \]

While for Rotational Motion,

\[ \text{disturbing torque} = \text{moment of inertia} \times \text{angular acceleration} \]

The description of motion is only meaningful if given relative to a frame of reference and it is important for the application of Newtons Second Laws of Motion that this frame be inertial. For the purpose of this work which involves short term navigation and control analysis it will be assumed that the Earth is such a reference frame, or more formally

**Assumption 1:** The Earth is considered fixed in space and thus an inertial reference frame.

For the general equations of motion it is also common to assume;

**Assumption 2:** The Airframe is a Rigid Body.

**Assumption 3:** The Mass, and Mass Distribution, of the Aircraft remain constant.

**Assumption 4:** The Aircraft is symmetrical about the \( XZ \) plane.

This allows the aircraft’s motion to be described by translation and rotation about the aircrafts centre of mass and for the mass and moments of inertia to remain constant. Products of Inertia can be assumed zero given the body axes of the aircraft are orientated as shown in Figure A.1, where plane \( o_{xb} - o_{z_b} \) forms a plane of symmetry. This leads to the general equations of motion

\[
\begin{align*}
X &= m(\ddot{U} - RV + QW) \\
Y &= m(\ddot{V} - PW + RU) \\
Z &= m(\ddot{W} - QU + PV) \\
L &= I_x \ddot{P} - (I_y - I_z)QR - I_{xz}(PQ + \dot{R}) \\
M &= I_y \ddot{Q} + (I_x - I_z)PR + I_{xz}(P^2 - R^2) \\
N &= I_z \ddot{R} - (I_x - I_y)PQ + I_{xz}(QR - \dot{P})
\end{align*}
\] (A.1)
Where

\[
X, Y, Z \quad - \quad \text{Force acting along Body Axes}
\]
\[
L, M, N \quad - \quad \text{Moments acting about Body Axes}
\]
\[
U, V, W \quad - \quad \text{Velocity expressed in Body Axes}
\]
\[
P, Q, R \quad - \quad \text{Angular Rates about Body Axes}
\]
\[
I_x, I_y, I_z \quad - \quad \text{Moment of Inertia about Body Axes}
\]
\[
I_{xz} \quad - \quad \text{Product of Inertia}
\]
\[
\Theta, \Phi, \Psi \quad - \quad \text{Euler Angles}
\]

The forces and moments acting upon the airframe can be expressed as the summation of their individual components. For this work it is assumed that these components are Aerodynamic \((a)\), Gravitational \((g)\), Aerodynamic Controls \((c)\), Power Effects \((p)\) and Atmospheric Disturbances \((d)\). Mathematically this is represented as (using component of force \(X\) as an example),

\[
X = X_a + X_g + X_c + X_p + X_d
\]

Before aircraft motion can be described in the inertial frame however, body velocities \(U, V \& W\) must be related to velocities in the inertial frame, \(\dot{X}, \dot{Y} \& \dot{Z}\). This is achieved through the use of Euler Angles which relate the body axes orientation with respect to the inertial reference frame, commonly referred to as Pitch (\(\Theta\)), Roll (\(\Phi\)) and Yaw (\(\Psi\)). Order of rotation is important as angular rotations are non-commutative, with the common convention adopted here of Yaw, followed by Pitch, followed by Roll. Mathematically the transformation from Body to Inertial Velocities is given as

\[
\begin{bmatrix}
\dot{X} \\
\dot{Y} \\
\dot{Z}
\end{bmatrix} =
\begin{bmatrix}
C_\Theta C_\Psi & S_\Theta S_\Phi C_\Psi - C_\Phi S_\Psi & S_\Theta C_\Phi C_\Psi + S_\Phi S_\Psi \\
C_\Theta S_\Psi & S_\Theta S_\Phi S_\Psi + C_\Phi C_\Psi & S_\Theta C_\Phi S_\Psi - S_\Phi C_\Psi \\
-S_\Theta & C_\Theta S_\Phi & C_\Theta C_\Phi
\end{bmatrix}
\begin{bmatrix}
U \\
V \\
W
\end{bmatrix}
\]  
(A.2)

Where \(S_\phi, (C_\phi)\) represents \(\sin \phi, (\cos \phi)\), respectively. This requires knowledge of the Euler Angles which are themselves a function of time. Through similar derivation of Euler Angles, Euler Rates (\(\dot{\Theta}, \dot{\Phi} \& \dot{\Psi}\)) can be expressed in terms of
Body Angular Rates \((P, Q & R)\), and is given as follows

\[
\begin{align*}
\dot{\Theta} &= Q \cos \Phi - R \sin \Phi \\
\dot{\Phi} &= P + Q \tan \Theta \sin \Phi + R \tan \Theta \cos \Phi \\
\dot{\Psi} &= (Q \sin \Phi + R \cos \Phi) \sec \Theta
\end{align*}
\] 

(Equation A.3)

Equally, Body Angular Rates can be expressed in terms of Euler Rates as follows

\[
\begin{align*}
P &= \dot{\Phi} - \dot{\Psi} \sin \Theta \\
Q &= \dot{\Theta} \cos \Phi + \dot{\Psi} \cos \Theta \sin \Phi \\
R &= \dot{\Psi} \cos \Theta \cos \Phi - \dot{\Theta} \sin \Phi
\end{align*}
\] 

(Equation A.4)

commonly referred to as the Airplane Kinematic Equations [216] or Auxiliary Equations of Motion [209].

With Euler Angles defined, it is possible to resolve the components of Force and Moments due to gravity \((X_g, Y_g, Z_g, L_g, M_g, N_g)\) into the Body Frame. Given the origin of the Body Axis System is defined at the aircraft’s c.g. there will be no moments due to gravity, thus \(L_g, M_g, N_g = 0\).

Assuming a flat earth model, components of force as a result of gravity can be expressed as,

\[
\begin{align*}
X_g &= -mg \sin \Theta \\
Y_g &= mg \cos \Theta \sin \Phi \\
Z_g &= mg \cos \Theta \cos \Phi
\end{align*}
\] 

(Equation A.5)

Substituting Equations (A.5) into (A.1) yields the 6 DOF Non-linear Equations of Motion,

\[
\begin{align*}
X &= m(\dot{U} - R \dot{V} + Q \dot{W} + g \sin \Theta) \\
Y &= m(\dot{V} - P \dot{W} + R \dot{U} - g \cos \Theta \sin \Phi) \\
Z &= m(\dot{W} - Q \dot{U} + P \dot{V} - g \cos \Theta \cos \Phi) \\
L &= I_x \dot{P} + (I_z - I_y)QR - I_{xz}(PQ + \dot{R}) \\
M &= I_y \dot{Q} + (I_x - I_z)PR + I_{xz}(P^2 - R^2)
\end{align*}
\] 

(Equation A.6)
\[ N = I_z \dot{R} - (I_x - I_y)PQ + I_{xz}(QR - \dot{P}) \]
\[ P = \Phi - \Psi \sin \Theta \]
\[ Q = \dot{\Theta} \cos \Phi + \dot{\Psi} \cos \Theta \sin \Phi \]
\[ R = \dot{\Psi} \cos \Theta \cos \Phi - \dot{\Theta} \sin \Phi \]

Equations (A.6) are non-linear and their solution by analytical means is not generally practical [210]. In addition, terms on the left hand side of the equation are yet to be replaced with suitable expressions, and these can be particularly difficult for generalised motion. It’s at this point that it is common to linearise the generalised equations of motion for small perturbations about a trim condition.

### A.2.1 Linearisation

To linearise the equations of motion, aircraft motion is assumed to be described by a mean motion, which represents the equilibrium or trimmed flight condition, and dynamic motion that accounts for perturbations about the mean motion. It’s important to note that this method will only hold for small perturbations about the mean motion, leading to,

**Assumption 5:** Aircraft motion is assumed to be comprised of a mean motion, denoted subscript zero (\(0\)), and dynamic motion caused by small perturbations, denoted by lower case.

The nine motion variables are then expressed as,

\[ U \triangleq U_0 + u \quad P \triangleq P_0 + p \quad \Theta \triangleq \Theta_0 + \theta \]
\[ V \triangleq V_0 + v \quad Q \triangleq Q_0 + q \quad \Phi \triangleq \Phi_0 + \phi \]
\[ W \triangleq W_0 + w \quad R \triangleq R_0 + r \quad \Psi \triangleq \Psi_0 + \psi \quad (A.7) \]

Similarly, expressions for force and moment become,

\[ X \triangleq X_0 + x \quad L \triangleq L_0 + l \]
\[ Y \triangleq Y_0 + y \quad M \triangleq M_0 + m_1 \]
\[ Z \triangleq Z_0 + z \quad N \triangleq N_0 + n \quad (A.8) \]
Moment perturbation of $M$ in this case is given the subscript $m_1$ to avoid confusion with aircraft mass, $m$. Replacing motion variables in equation (A.6) with small perturbation variables results in,

$$
X_0 + x = m \left[ (\dot{U}_0 + \dot{u}) - (R_0 + r)(V_0 + v) + (Q_0 + q)(W_0 + w) \ldots \right. \\
\left. + g \sin (\Theta_0 + \theta) \right] \\
Y_0 + y = m \left[ (\dot{V}_0 + \dot{v}) - (P_0 + p)(W_0 + w) + (R_0 + r)(U_0 + u) \ldots \\
\left. - g \cos (\Theta_0 + \theta) \sin (\Phi_0 + \phi) \right] \\
Z_0 + z = m \left[ (\dot{W}_0 + \dot{w}) - (Q_0 + q)(U_0 + u) + (P_0 + p)(V_0 + v) \ldots \\
\left. - g \cos (\Theta_0 + \theta) \cos (\Phi_0 + \phi) \right] \\
L_0 + l = I_x (\dot{P}_0 + \dot{p}) + (I_x - I_y)(Q_0 + q)(R_0 + r) \ldots \\
M_0 + m_1 = I_y (\dot{Q}_0 + \dot{q}) + (I_x - I_z)(P_0 + p)(R_0 + r) \ldots \\
\left. + I_{xz}((P_0 + p)^2 - (R_0 + r)^2) \right] \\
N_0 + n = I_z (\dot{R}_0 + \dot{r}) - (I_x - I_y)(P_0 + p)(Q_0 + q) \ldots \\
\left. + I_{xz}((Q_0 + q)(R_0 + r) - (\dot{P}_0 + \dot{p}) \right] \\
P_0 + p = (\dot{\Phi}_0 + \dot{\phi}) - (\dot{\Psi}_0 + \dot{\psi}) \sin (\Theta_0 + \theta) \\
Q_0 + q = (\dot{\Theta}_0 + \dot{\theta}) \cos (\Phi_0 + \phi) \ldots \\
\left. + (\dot{\Psi}_0 + \dot{\psi}) \cos (\Theta_0 + \theta) \sin (\Phi_0 + \phi) \right] \\
R_0 + r = (\dot{\Psi}_0 + \dot{\psi}) \cos (\Theta_0 + \theta) \cos (\Phi_0 + \phi) \\
\left. - (\dot{\Theta}_0 + \dot{\theta}) \sin (\Phi_0 + \phi) \right]
$$

Recognising that mean motion variables are constants, derivatives of such terms go to zero, i.e. $\dot{X}_0 = 0$. Given that perturbation variables are by definition small, it can also be assumed that the product or square of such terms can be neglected, leading to the sixth assumption;

**Assumption 6:** The magnitude of perturbations is assumed sufficiently small such that products and squares of such terms are considered negligible. Furthermore, small angle approximations are made such that $\cos \alpha \approx 1$, $\sin \alpha \approx \alpha$ and $\tan \alpha \approx \alpha$. 
A.2 Aircraft Model

Introducing these approximations to Equations (A.9) simplifies the equations to the following,

\[ X_0 + x = m\left[\dot{u} + Q_0W_0 + Q_0w + gW_0 - R_0V_0 - R_0v \ldots + g\left(\sin \Theta_0 + \theta \cos \Theta_0\right)\right] \]

\[ Y_0 + y = m\left[\dot{v} + R_0U_0 + R_0u + rU_0 - P_0W_0 - P_0w - pW_0 \ldots - g\left(\cos \Theta_0 \sin \Phi_0 + \phi \cos \Theta_0 \cos \Phi_0 - \theta \sin \Theta_0 \sin \Phi_0\right)\right] \]

\[ Z_0 + z = m\left[\dot{w} + P_0V_0 + P_0v + pV_0 - Q_0U_0 - Q_0u - qU_0 \ldots - g\left(\cos \Theta_0 \cos \Phi_0 - \theta \sin \Theta_0 \cos \Phi_0 - \phi \cos \Theta_0 \sin \Phi_0\right)\right] \]

\[ L_0 + l = I_x\dot{p} + (I_z - I_y)(Q_0R_0 + Q_0r + qR_0) \ldots - I_xz(P_0Q_0 + P_0q + pQ_0 + \dot{r}) \]

\[ M_0 + m_1 = I_y\dot{q} + (I_x - I_z)(P_0R_0 + P_0r + pR_0) \ldots + I_xz(P_0^2 + 2(P_0p - R_0r) - R_0^2) \]

\[ N_0 + n = I_x\dot{r} - (I_x - I_y)(P_0Q_0 + P_0q + pQ_0) \ldots + I_xz(Q_0R_0 + Q_0r + qR_0 - \dot{p}) \]

\[ P_0 + p = \dot{\Phi}_0 + \dot{\phi} - \dot{\Psi}_0 + \dot{\psi} \sin \Theta_0 - \theta \dot{\Phi}_0 \cos \Theta_0 \]

\[ Q_0 + q = (\dot{\Theta}_0 + \dot{\theta}) \cos \Phi_0 + \dot{\Psi}_0 \cos \Theta_0 \sin \Phi_0 - \theta \dot{\Psi}_0 \sin \Theta_0 \sin \Phi_0 \]

\[ R_0 + r = \dot{\Phi}_0 \cos \Theta_0 \cos \Phi_0 - (\dot{\Theta}_0 + \dot{\theta}) \sin \Phi_0 - \theta \dot{\Phi}_0 \sin \Theta_0 \cos \Phi_0 \]

\[ l = I_x\dot{p} + (I_z - I_y)(Q_0r + qR_0) - I_xz(P_0q + pQ_0 + \dot{r}) \] (A.11)

Setting perturbations to zero then allows equations to be solved for trimmed flight that once subtracted from (A.10) yields the equations for perturbations,
Mathematical Background

\[ m_1 = I_y \dot{q} + (I_x - I_z)(P_0 r + p R_0) + 2I_{xz}(P_0 p - R_0 r) \]
\[ n = I_z \dot{r} - (I_x - I_y)(P_0 q + p Q_0) + I_{xz}(Q_0 r + q R_0 - \dot{p}) \]
\[ p = \dot{\phi} - \theta \dot{\Phi}_0 \cos \Theta_0 - \dot{\psi} \sin \Theta_0 \]
\[ q = \dot{\theta} \cos \Phi_0 - \theta \dot{\Psi}_0 \sin \Theta_0 \sin \Phi_0 \ldots \]
\[ + \dot{\phi} (\dot{\Psi}_0 \cos \Theta_0 \cos \Phi_0 - \dot{\Theta}_0 \sin \Phi_0) \ldots \]
\[ + \dot{\psi} \cos \Theta_0 \sin \Phi_0 \]
\[ r = \dot{\psi} \cos \Theta_0 \cos \Phi_0 - \dot{\theta} \sin \Phi_0 - \theta \dot{\Psi}_0 \sin \Theta_0 \cos \Phi_0 \ldots \]
\[ - \dot{\phi} (\dot{\Psi}_0 \sin \Phi_0 + \dot{\Theta}_0 \cos \Phi_0) \]

A.2.2 Trimmed Flight

Before the linearised equations of motion given by (A.11) can be used, a trimmed flight condition must be chosen. Considering the simplest case of an aircraft already tracking a feature, it can be assumed that the aircraft is trimmed for Straight, Wings Level, Symmetric flight, resulting in

\[ \dot{\Theta}_0 = \dot{\Phi}_0 = \dot{\Psi}_0 = 0 \quad \Phi_0 = 0 \quad V_0 = 0 \]

As Euler Rates are zero so will angular rates \( P_0 = Q_0 = R_0 = 0 \). Heading angle \( \Psi_0 \) is also arbitrary and can be set to zero, reducing the linearised equations of motion to,

\[ x = m(\dot{u} + q W_0 + g \theta \cos \Theta_0) \]
\[ y = m(\dot{v} + r U_0 - p W_0 - g \phi \cos \Theta_0) \]
\[ z = m(\dot{w} - q U_0 + g \theta \sin \Theta_0) \]
\[ l = I_x \dot{p} - I_{xz} \dot{r} \]
\[ m_1 = I_y \dot{q} \]
\[ n = I_z \dot{r} - I_{xz} \dot{p} \]
\[ p = \dot{\phi} - \psi \sin \Theta_0 \]
\[ q = \dot{\theta} \]
\[ r = \dot{\psi} \cos \Theta_0 \]
A.2 Aircraft Model

For this particular trim condition it can be seen that the equations are now simplified to the extent that two independent sets of equations can be formed, one a function of \( U, W, Q & \Theta \), the other a function of \( V, P, R, \Phi & \Psi \). These are commonly referred to as the decoupled equations of motion as they describe longitudinal and lateral motion respectively. Given the controller design is only intended to control lateral motion of the aircraft, longitudinal equations can be removed,

\[
\begin{align*}
  y &= m(\dot{v} + U_0 r - W_0 p - g\phi \cos \Theta_0) \\
  l &= I_x \dot{p} - I_{xx} \dot{r} \\
  n &= I_z \dot{r} - I_{xz} \dot{p} \\
  p &= \dot{\phi} - \dot{\psi} \sin \Theta_0 \\
  r &= \dot{\psi} \cos \Theta_0
\end{align*}
\]

(A.12)

A.2.3 Aerodynamic Terms

With the right hand side of the equations developed, attention can turn to the left hand side. Authors take varying approaches at this point but in general it’s common to use stability derivatives. It will be assumed that the Aerodynamic Force & Moments acting on the Aircraft are dependent on the disturbed motion variables & their derivatives. Given disturbed motion variables are small, only the first term of each series is considered significant, up to the first derivative of higher order motion variables.

Mathematically, this is expressed as the sum of a Taylor series, with each series involving one motion variable or the first derivative of a motion variable. Taking for example the lateral equations of motion, the aerodynamic components of the motion variable \( y \) \((y_a, y_c)\) become,

\[
\begin{align*}
  y_a &= \frac{\partial y}{\partial v} \dot{v} + \frac{\partial y}{\partial \dot{v}} \ddot{v} + \frac{\partial y}{\partial p} \dot{p} + \frac{\partial y}{\partial \dot{p}} \ddot{p} + \frac{\partial y}{\partial r} \dot{r} + \frac{\partial y}{\partial \dot{r}} \ddot{r} \\
  y_c &= \frac{\partial y}{\partial \delta_a} \delta_a + \frac{\partial y}{\partial \dot{\delta}_a} \dot{\delta}_a + \frac{\partial y}{\partial \delta_r} \delta_r + \frac{\partial y}{\partial \dot{\delta}_r} \dot{\delta}_r
\end{align*}
\]

Fortunately, for conventional airframes many derivatives have been found through testing to be negligible [209], for example in terms of lateral motion, the following derivatives are generally considered negligible,
\[
\frac{\partial y}{\partial p}, \frac{\partial y}{\partial r}, \frac{\partial y}{\partial \dot{v}}, \frac{\partial y}{\partial \delta_a}, \frac{\partial y}{\partial \delta_r}
\]
\[
\frac{\partial l}{\partial \dot{v}}, \frac{\partial l}{\partial \delta_a}, \frac{\partial l}{\partial \delta_r}, \frac{\partial l}{\partial \dot{p}}, \frac{\partial l}{\partial \dot{r}}, \frac{\partial l}{\partial \delta_a}, \frac{\partial l}{\partial \delta_r}
\]
\[
\frac{\partial n}{\partial \dot{v}}, \frac{\partial n}{\partial \delta_a}, \frac{\partial n}{\partial \delta_r}, \frac{\partial n}{\partial \dot{p}}, \frac{\partial n}{\partial \dot{r}}, \frac{\partial n}{\partial \delta_a}, \frac{\partial n}{\partial \delta_r}
\]

This simplifies the aerodynamic terms considerably, leaving,

\[
y = \frac{\partial y}{\partial v} v + \frac{\partial y}{\delta_a} \delta_a + \frac{\partial y}{\delta_r} \delta_r
\]
\[
l = \frac{\partial l}{\partial v} v + \frac{\partial l}{\dot{p}} p + \frac{\partial l}{\dot{r}} r + \frac{\partial l}{\delta_a} \delta_a + \frac{\partial l}{\delta_r} \delta_r
\]
\[
n = \frac{\partial n}{\partial v} v + \frac{\partial n}{\dot{p}} p + \frac{\partial n}{\dot{r}} r + \frac{\partial n}{\delta_a} \delta_a + \frac{\partial n}{\delta_r} \delta_r
\]

Common notation concerning stability derivatives is adopted, defining,

\[
Y_v \triangleq \frac{1}{m} \frac{\partial y}{\partial v}
\]
\[
L_v \triangleq \frac{1}{I_x} \frac{\partial l}{\partial v}
\]
\[
N_v \triangleq \frac{1}{I_z} \frac{\partial n}{\partial v}
\]

Finally aerodynamic terms for lateral motion can be expressed as

\[
y = m(Y_v v + Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r)
\]
\[
l = I_x(L_v v + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r)
\]
\[
n = I_z(N_v v + N_{\delta_a} \delta_a + N_{\delta_r} \delta_r)
\]  

(A.13)

The remaining terms left to consider are those due to power and atmospheric disturbances. Initially it will be assumed that the aircraft is operating in a disturbance free atmosphere, thus \(y_d, l_d, n_d\) would be zero. Conventional power plants generally have little influence on the lateral motion of an aircraft and thus the power terms \(y_p, l_p, n_p\) would be considered negligible.
With these terms set to zero, the left hand side of equation (A.12) can be re-arranged with respect to derivative terms such that,

\[ \dot{v} = Y_v v + W_0 p - U_0 r + g \phi \cos \Theta_0 + Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r \]
\[ \dot{p} - \left( \frac{I_{xz}}{I_z} \right) \dot{r} = L_v v + L_p p + L_r r + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r \]
\[ \dot{r} - \left( \frac{I_{xz}}{I_z} \right) \dot{p} = N_v v + N_p p + N_r r + N_{\delta_a} \delta_a + N_{\delta_r} \delta_r \]
\[ \dot{\phi} = p + (\tan \Theta_0) r \]
\[ \dot{\psi} = (\sec \Theta_0) r \]

These are now in such a way that the equations can be expressed in state space form. It should be noted that boldfaced lower case characters will be used to represent vectors, while upper case boldfaced characters will be used to represent matrices.

\[ \dot{x}(t) = M^{-1} A' x(t) + M^{-1} B' u(t) \]  
(A.14)

where,

\[ x(t) = \begin{bmatrix} v \\ p \\ r \\ \phi \\ \psi \end{bmatrix}, \quad A' = \begin{bmatrix} Y_v & W_0 & -U_0 & gC\Theta_0 & 0 \\ L_v & L_p & L_r & 0 & 0 \\ N_v & N_p & N_r & 0 & 0 \\ 0 & 1 & T_{\Theta_0} & 0 & 0 \\ 0 & 0 & S_{\Theta_0} & 0 & 0 \end{bmatrix} \]

\[ M^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{-I_{xz}}{I_z} & 0 & 0 \\ 0 & \frac{I_{xz}}{I_z} & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \]

\[ B' = \begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \]

\[ u(t) = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \]
Stability Derivatives

The final step in the process is calculating values for the stability derivatives. These are a function of the aircraft under consideration, where the aircraft aero-dynamic parameters are commonly expressed as stability coefficients, related to the stability derivatives as follows,

\[ Y_v = \frac{QS}{m} C_{y_v} \]
\[ L_v = \frac{QSb}{I_x} C_{l_v} \]
\[ N_v = \frac{QSb}{I_z} C_{n_v} \]
\[ Y_{\delta_a} = \frac{QS}{m} C_{y_{\delta_a}} \]
\[ L_p = \frac{QSb^2}{2I_x U_0} C_{l_p} \]
\[ N_p = \frac{QSb^2}{2I_z U_0} C_{n_p} \]
\[ Y_{\delta_r} = \frac{QS}{m} C_{y_{\delta_r}} \]
\[ L_r = \frac{QSb^2}{2I_x U_0} C_{l_r} \]
\[ N_r = \frac{QSb^2}{2I_z U_0} C_{n_r} \]
\[ L_{\delta_a} = \frac{QSb}{I_x} C_{l_{\delta_a}} \]
\[ N_{\delta_a} = \frac{QSb}{I_z} C_{n_{\delta_a}} \]
\[ L_{\delta_r} = \frac{QSb}{I_x} C_{l_{\delta_r}} \]
\[ N_{\delta_r} = \frac{QSb}{I_z} C_{n_{\delta_r}} \]

Where \( Q \) is the dynamic pressure, given by

\[ Q = \frac{1}{2} \rho V_P^2 \tag{A.15} \]

\( \rho \) is air density and \( V_P \) the magnitude of total velocity expressed as

\[ V_P = \sqrt{(U_0 + u)^2 + (V_0 + v)^2 + (W_0 + w)^2} \tag{A.16} \]

It is important to note that the coefficients are typically provided in Aircraft Stability or Wind Axes, as apposed to Body Axes. For the general case of trimmed symmetric flight (\( \beta_0 = 0 \)) the aircraft stability axes are simply a rotation of the body axes rotated about \( oy_b \) by the angle of attack, otherwise referred to as the body incidence angle, \( \alpha_0 \). Thus the plane of symmetry \( ox_b - oz_0 \) remains, while \( oy_b \) and \( oy_w \) coincide. For a stability axis system,

\[ U_{0_w} = V_P \]
\[ V_{0_w} = 0 \]
\[ W_{0_w} = 0 \]
\[ \alpha_{0_w} = 0 \]
Euler Angle, $\Theta_{0_w}$, angle of attack, $\alpha_{0_w}$, and steady state flight path angle, $\gamma_0$, are all related by

$$\Theta_{0_w} = \gamma_0 + \alpha_{0_w}$$

thus if a horizontal flight path is assumed during then $\gamma_0 = 0$ and

$$\Theta_{0_w} = 0;$$

Adopting a stability axis system and applying these approximations to the state space representation of equation A.14 allows lateral motion to then be described by,

$$\begin{bmatrix}
\dot{v} \\
\dot{p} \\
\dot{r} \\
\dot{\phi}
\end{bmatrix} =
\begin{bmatrix}
Y_v & 0 & -V_P & g \\
L'_v & L'_p & L'_r & 0 \\
N'_a & N'_p & N'_r & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
v \\
p \\
r \\
\phi
\end{bmatrix}
+ 
\begin{bmatrix}
Y_{\delta_a} & Y_{\delta_r} \\
L'_{\delta_a} & L'_{\delta_r} \\
N'_{\delta_a} & N'_{\delta_r} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_a \\
\delta_r
\end{bmatrix}
$$

where,

$$L'_\epsilon = L_\epsilon + \frac{I_{xz}}{I_x}N'_\epsilon \\
N'_\epsilon = N'_\epsilon + \frac{I_{xz}}{I_z}L'_\epsilon \\
L'_\epsilon = \frac{L_\epsilon}{1 - I_{xz}(I_x I_z)^{-1}}$$

A similar derivation can be taken for longitudinal motion [209], that allows longitudinal motion to be expressed as,

$$\begin{bmatrix}
\dot{u} \\
\dot{w} \\
\dot{q} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
X_u & X_w & 0 & -g \\
Z_u & Z_w & V_P & 0 \\
M'_u & M'_w & M'_q & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
u \\
w \\
q \\
\theta
\end{bmatrix}
+ 
\begin{bmatrix}
X_{\delta_u} & X_{\delta_{\theta}} \\
Z_{\delta_u} & Z_{\delta_{\theta}} \\
M'_{\delta_u} & M'_{\delta_{\theta}} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_u \\
\delta_w \\
\delta_q \\
\delta_{\theta}
\end{bmatrix}
$$

where,

$$M'_\epsilon = M_\epsilon + M_\dot{\epsilon}Z_\epsilon \\
M'_q = M_q + M_\dot{\epsilon}V_P$$

The following provides an example of calculating stability derivatives for a given flight condition.
A.2.4 Aerosonde® Model

Parameters for the Aerosonde are given as follows,

- Mass (Gross) - 13.5kg
- Wing Span (\(b\)) - 2.90m
- Wing Area (\(S\)) - 0.550m\(^2\)
- Mean Aerodynamic Chord (\(MAC\)) - 0.190m
- Inertia Components (\(I_x, I_z, I_{xz}\)) - 0.824, 1.76, 0.120 kgm\(^2\)

where Lateral Aerodynamic Coefficients used in calculating the stability derivatives, are given as,

\[
\begin{align*}
C_{Y_\beta} &= -0.83 & C_{L_\beta} &= -0.13 & C_{N_\beta} &= 0.0726 \\
C_{Y_{ba}} &= -0.075 & C_{L_{ba}} &= -0.1695 & C_{N_{ba}} &= 0.0108 \\
C_{Y_{sr}} &= 0.1914 & C_{L_{sr}} &= 0.0024 & C_{N_{sr}} &= -0.0693 \\
C_{L_p} &= -0.5051 & C_{N_p} &= -0.069 \\
C_{L_r} &= 0.2519 & C_{N_r} &= -0.0946
\end{align*}
\]

Note coefficients \(C_{Y_v}, C_{L_v}, C_{N_v}\) are expressed in terms of sideslip angle \(\beta\) hence \(C_{Y_\beta}, C_{L_\beta}, C_{N_\beta}\). Recognising that

\[
\beta = \arctan\left(\frac{V_0 + v}{V_P}\right)
\]

Initial sideslip of zero and assuming small angles

\[
\beta \approx \frac{v}{V_P}
\]

Taking the partial derivative with respect to \(v\)

\[
\frac{\partial \beta}{\partial v} = \frac{\partial}{\partial v} \frac{v}{V_P}
\]
From equation (A.16) it can be seen that $V_P$ is a function of $v$, although recalling products of perturbations are assumed small,

\[ V_P = \sqrt{(U_0 + u)^2 + v^2 + w^2} \approx U_0 \sqrt{1 + \frac{2u}{U_0}} \]

Considering $u \ll U_0$ then $V_P \approx U_0$, such that,

\[ \frac{\partial \beta}{\partial v} = \frac{1}{V_P} \]

Terms $Y_v$, $L_v$, $N_v$ can then be expressed as a function of $C_{Y\beta}$, $C_{L\beta}$, $C_{N\beta}$,

\[ Y_v = \frac{QS}{mV_P} C_{y\beta} \]
\[ L_v = \frac{QSb}{I_x V_P} C_{l\beta} \]
\[ N_v = \frac{QSb}{I_z V_P} C_{n\beta} \]

Finally stability derivatives can be calculated for a given airspeed. Taking a slow cruise speed for the Aerosonde airframe of 70 km/h (19.4 m/s), dynamic pressure can then be calculated from (A.15) as $Q = 230.5$ Pa, where density is taken at sea level given the low operating conditions. Finally the linearised equations of motion for lateral dynamics can be constructed from (A.17),

\[
\begin{bmatrix}
\dot{v} \\
\dot{p} \\
\dot{r} \\
\dot{\phi} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
-0.402 & 0 & -19.4 & 9.81 & 0 \\
-2.90 & -17.1 & 8.27 & 0 & 0 \\
0.583 & -2.25 & -0.914 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
v \\
p \\
r \\
\phi \\
\psi
\end{bmatrix} +
\begin{bmatrix}
-0.704 & 1.80 \\
-76.0 & -1.06 \\
-2.95 & -14.6 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_a \\
\delta_r
\end{bmatrix}
\]

describing lateral motion about a steady straight, wings level, symmetric trim condition. This can then be used in the development of control where motion can be approximated as remaining close to the operating point.
A.3 Camera Model

A common model for imaging sensors used in computer vision and robotics is the pin hole camera. Under this model, the mathematical relationship between the image plane and camera frame can be expressed as follows

\[
\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & \tau & c_x & 0 \\ 0 & \eta f & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}
\]

(A.19)

Coordinates \((u, v, 1)\) and \((x_c, y_c, z_c, 1)\) refer to the image and camera frames respectively, expressed in homogeneous coordinates. The term \(\lambda\) is the homogeneous scale factor, in this case equal to \(z_c\); \(f\) is the focal length of the camera, where a fixed focal length lens is assumed; coordinates \((c_x, c_y)\) specify the principal point that relates the image centre to that of the optical centre; and parameters \(\eta\) & \(\tau\) account for the non-uniform pixel characteristics aspect ratio and skew, respectively, although it will be assumed in this work that the sensor has uniform pixels, hence \(\eta = 1\) and \(\tau = 0\).

A.4 Interaction Matrix

In principle, an interaction matrix expresses the velocity of image features as a function of camera velocity. For a set of features, \(s\), the velocity of features as seen in the image plane is expressed as a function of camera velocity screw, \(T_c\), by the interaction matrix as follows,

\[
\dot{s} = L_s T_c
\]

For a camera with 6 degrees-of-freedom \(T_c\) can be expressed as

\[
T_c = \begin{bmatrix} U_c & V_c & W_c & P_c & Q_c & R_c \end{bmatrix}^T
\]

Deriving the interaction matrix for a point feature forms the foundation for a number of other features, including line features, and is thus included in the following section.
A.4.1 Point Feature

Consider a fixed point feature $P$ located at $P(P_x, P_y, P_z)$ in the camera frame, where the projected location in the image plane will be denoted $(p_x, p_y)$. As the feature is assumed to be fixed within the camera frame, any motion of the projected feature will be as a result of camera motion, or more specifically, relative velocity generated by a moving, rotating, reference frame. Relating such motion to $(p_x, p_y)$ is simplified if motion of the camera frame is expressed as the equivalent motion of a feature moving within a stationary frame. This can be achieved by first recognising that 6 DOF motion can be decoupled into translational, $V_T = T_c(U_c, V_c, W_c)$, and rotational motion, $V_\omega = T_c(P_c, Q_c, R_c)$.

Figure A.3a illustrates translational motion of a camera within a fixed frame $(x, y, z)$ over a small time period, $dt$. It can be seen that point $P$ will move with equal but opposite motion with respect to camera frame $(x_1, y_1, z_1)$, as illustrated in Figure A.3b. If the motion is considered from the camera frame, the equivalent motion of point $P$ is seen to be given by $\mathbf{d}\mathbf{r} = (-dx, -dy, -dz)$, dividing by $dt$ results in,

$$\frac{\mathbf{d}\mathbf{r}}{dt} = \left(-\frac{dx}{dt}, -\frac{dy}{dt}, -\frac{dz}{dt}\right)$$

$$\mathbf{P}(V_T) = -V_T$$
Equivalent motion of a feature due to a rotating reference frame is dependent on the radius of the point from the axis of rotation. When a camera frame rotates, a fixed point will appear from the perspective of the camera frame to translate a circle in a plane perpendicular to the rotation axis.

From Figure A.4 it can be seen that the equivalent rotation is equal but opposite in direction to rotation of the camera frame.

Recognising that \( dp \) is equal to,

\[
dp = rd\theta
\]

and given \( d\theta \) is small, the direction of vector \( dp \) can be approximated by,

\[
d\hat{p} = \frac{P \times V_\omega}{|P \times V_\omega|}
\]

thus,

\[
dp = \frac{r d\theta}{|P \times V_\omega|} P \times V_\omega
\]

and by definition we have,

\[
|P \times V_\omega| = |P||V_\omega| \sin \phi
\]
and from figure A.3b,\[ r = |P| \sin \phi \]

term \( dp \) becomes,\[ dp = \frac{d\theta}{|V_\omega|} P \times V_\omega \]

and as,\[ |V_\omega| = \frac{d\theta}{dt} \]

finally we have,

\[
\frac{dp}{dt} = P \times V_\omega \\
\dot{P}(V_\omega) = -V_\omega \times P
\]

Combining these results produces an expression for velocity of a moving point in a stationary frame equivalent to that of a stationary point observed from a moving, rotating, reference frame,

\[
\dot{P}(V_T, V_\omega) = -(V_T + V_\omega \times P)
\] (A.20)

this can then be expressed in terms of the point feature’s velocity in the image frame. Beginning with an expansion of (A.20),

\[
\dot{P}_x = -U_c + R_c P_y - Q_c P_z \\
\dot{P}_y = -V_c - R_c P_x + P_c P_z \\
\dot{P}_z = -W_c + Q_c P_x - P_c P_y
\] (A.21)

Recalling the relationship of (A.19) the image plane coordinates can be related to the camera frame through the relationship,

\[
p_x = f \left( \frac{P_x}{P_z} \right) + c_x \\
p_y = f \left( \frac{P_y}{P_z} \right) + c_y
\] (A.22)

then differentiating with respect to time,

\[
\dot{p}_x = \frac{f}{P_z^2} (\dot{P}_x P_z - P_x \dot{P}_z) \\
\dot{p}_y = \frac{f}{P_z^2} (\dot{P}_y P_z - P_y \dot{P}_z)
\]
and substituting into equation (A.21),

$$
\dot{p}_x = -\left(\frac{f}{P_z}\right) U_c + f \left(\frac{P_x}{P_z}^2\right) W_c + f \left(\frac{P_x P_y}{P_z^2}\right) P_c - f \left(1 + \left(\frac{P_x}{P_z}\right)^2\right) Q_c + f \left(\frac{P_y}{P_z}\right) R_c
$$

$$
\dot{p}_y = -\left(\frac{f}{P_z}\right) V_c + f \left(\frac{P_y}{P_z}^2\right) W_c + f \left(1 + \left(\frac{P_y}{P_z}\right)^2\right) P_c - f \left(\frac{P_x P_y}{P_z^2}\right) Q_c - f \left(\frac{P_x}{P_z}\right) R_c
$$

Although this expresses image feature velocity as a function of camera velocity, it still relies on the known position of the feature with respect to the camera frame. This can be reduced by recalling (A.22),

$$
P_x \frac{P}{P_z} = \frac{1}{f} (p_x - c_x)
$$

$$
P_y \frac{P}{P_z} = \frac{1}{f} (p_y - c_y)
$$

resulting in,

$$
\dot{p}_x = -\left(\frac{f}{P_z}\right) U_c + \left(\frac{p_x - c_x}{P_z}\right) W_c + \left(\frac{(p_x - c_x)(p_y - c_y)}{f}\right) P_c - f \left(1 + \left(\frac{p_x - c_x}{f}\right)^2\right) Q_c + (p_y - c_y) R_c
$$

$$
\dot{p}_y = -\left(\frac{f}{P_z}\right) V_c + \left(\frac{p_y - c_y}{P_z}\right) W_c + f \left(1 + \left(\frac{p_y - c_y}{f}\right)^2\right) P_c - \left(\frac{(p_x - c_x)(p_y - c_y)}{f}\right) Q_c - (p_x - c_x) R_c
$$

If an ideal camera model is assumed, with \((c_x, c_y)\) at \((0, 0)\), then the equations can be further reduced and formed into matrices,

$$
\begin{bmatrix}
\dot{p}_x \\
\dot{p}_y
\end{bmatrix} =
\begin{bmatrix}
-\frac{f}{P_z} & 0 & \frac{p_x}{P_z} & \frac{p_x p_y}{f} & -f - \frac{p_x^2}{f} & p_y \\
0 & -\frac{f}{P_z} & \frac{p_y}{P_z} & \frac{p_y^2}{f} & -\frac{p_x p_y}{f} & -p_x
\end{bmatrix} T_c
\tag{A.23}
$$
which is of the form,

\[ \dot{s} = L_s T_c \]

where,

\[ T_c = \begin{bmatrix} U_c & V_c & W_c & P_c & Q_c & R_c \end{bmatrix}^T \]
The following section details the simulation environment developed over the course of this research that was used in the development and testing of the presented work. The simulation environment comprises of three main sections, aircraft simulation, synthetic imagery and control. The following provides a description of each of these systems.

### B.1 Simulink Environment

To assess the performance of developed techniques, the work presented in this thesis was simulated in a MATLAB Simulink® environment using AeroSim Blockset Version 1.2 provided free of charge for academic and non-commercial use by Unmanned Dynamics [214]. The blockset provides two nonlinear 6 DOF aircraft dynamic models, specifically the Navion general aviation airplane and the Aerosonde® UAV. A fixed-step continuous solver using fourth-order Runge-Kutta numerical integration was selected with 10 ms time steps to preserve simulation accuracy. Figure B.1 provides an overview of the simulation environment, where the inclusion of subsystems with dashed outlines is noted in specific chapters.
Figure B.1: Overview of Simulation Environment developed for testing and assessment of controllers.
B.1 Simulink Environment

B.1.1 Aircraft Model

The aircraft model chosen in this instance is based on the Aerosonde UAV shown in Figure B.2a that is one of two aircraft models included in the AeroSim blockset [214]. With a wingspan of 2.9 m and a total weight under 15 kg the platform is of a size that is likely to be chosen for civilian inspection tasks, with the additional benefit of providing over 10 hours endurance whilst carrying a full sensor payload.

![Figure B.2: UAV Models](image)

Although the simulation environment allows the numerical evaluation of the nonlinear equations of motion, it should be noted that the stability and control coefficients of the chosen model are constant. It is therefore important to consider the region over which the linear model is valid, as the accuracy of the simulation can be expected to decrease as the model is used to simulate motion away from the point of linearisation. This is particularly important when operating at high angles of attack or sideslip as these regions can be expected to introduce nonlinearities and, if not appropriately considered, could lead to an incorrect interpretation of results.

As the research focuses on the use of alternate manoeuvres with a large component relying on sideslip, a concern is raised over the models suitability at large angles of sideslip. The model itself defines limits of $\pm 28^\circ$ sideslip, although the accuracy of a linear model over this range is questionable given the associated coefficients are generally noted as difficult to estimate at large angles [210, 226]. Although the model can be linked back to an original simulation environment developed by Aerosonde North America, no data is available to validate the model.
at the stated limits of sideslip$^1$. Given access to the UAV is also not possible for verification through experiments, establishing reasonable limits over which the linear approximation can be applied was therefore further investigated.

General insight can be obtained from sources including the USAF Stability and Control Datcom that combine theoretical calculations with experimental data to provide semi-empirical methods for the estimation of coefficients [226]. Unfortunately the airframe under consideration has a number of unconventional design characteristics that limit the conclusions that can be drawn from this material. For this reason insight was sought through similar platforms for which data was available over a wide range of sideslip angles.

In 1991 Bray conducted a series of wind tunnel experiments in an effort to provide accurate modelling of the Pioneer Remotely Piloted Vehicle (RPV) for pilot simulation training in the US Navy [227]. Although the platform is larger in wingspan and weight (5.2 m, 190 kg), it shares a similar configuration to the Aerosonde with a shortened fuselage supporting a separate tail section via twin booms to accommodate a pusher style power plant as pictured in Figure B.2b. The work included scale model testing of sideslip over $\pm20^\circ$, estimating coefficients for side-force ($C_{Y_\beta}$), rolling moment ($C_{l_\beta}$) and yaw moment ($C_{n_\beta}$). The coefficients were found to become nonlinear outside a region of $\pm15^\circ$ sideslip, far inside the bounds set by the Aerosonde model of $\pm28^\circ$. For this reason the Aerosonde model used within the simulation environment would be further restricted to operating within bounds of $\pm15^\circ$ sideslip to ensure greater confidence in results.

A further point to note is the unconventional tail system employed by the Aerosonde that utilises a twin boom to support a V tail as opposed to a conventional T tail. In this configuration a pair of Ruddervators provide Elevator and Rudder style control, with rudder response generated through equal actuation of control surfaces, and opposing deflection initiating an elevator response. A valid argument may be raised over the extension of the results presented in the thesis to UAVs with conventional tails systems. Closer inspection however reveals that the model approximates the V tail as a conventional T tail, with separate stability derivatives for elevator and rudder with no terms for cross coupling. Therefore

$^1$Although not explicitly stated, M. Niculescu who founded Unmanned Dynamics discusses in [47] the adaption of an original simulation environment in use by Aerosonde North America for the Matlab Simulink environment, coinciding with the later release as the AeroSim blockset.
results can be expected to be equally applicable to UAVs with conventional tail systems.

B.1.2 Controllers

The controllers used within the simulation environment include basic functions that would be provided by an autopilot including:

- Altitude Hold, where a desired Rate-of-Climb (RoC) is generated proportional \( P = 0.15 \) to current altitude error, where RoC is regulated through Proportional-Integral (PI) control \( (P = 0.15, I = 0.015) \), commanding elevator limited to \( \pm 30^\circ \);

- Bearing Hold, controlled via desired bank angle generated by Proportional-Integral-Derivative (PID) control \( (P = 4, I = 0.005, D = 0.1) \), where bank angle is regulated through PID control \( (P = 0.07, I = 0.01, D = 0.01) \) commanding ailerons limited to \( \pm 45^\circ \);
  - Wings-Level Hold, utilising lower level bank angle control to maintain wings level flight \( \phi = 0^\circ \) that is required for the STT IBVS controller of Chapter 3. Gains are switched during these periods to improve performance \( (P = 0.07 \rightarrow 0.22, I = 0.01 \rightarrow 0.005, D = 0.01 \rightarrow 0.001) \).

- Airspeed Hold, regulated via Proportional-Integral (PI) control \( P = 0.5, I = 0.1 \) commanding throttle.

The autopilot controller is implemented to provide navigation to the inspection area at the beginning of the simulation and also provide longitudinal control in the form of airspeed and altitude hold for the FS IBVS controller, and the addition of wings-level hold for the STT controller.

A second controller is included in the form of a PBVS controller and is used to provide a generic comparison to techniques currently proposed for similar vision based tasks. The PBVS controller is developed on the principals of the ‘Good Helmsman’ guidance law developed by Rysdyk [83], commanding desired heading as a function of Cross Track Error, as measured between the aircraft and feature centreline. Cross track error in this instance is actually measured directly based on known aircraft position and features, thus providing an ideal case of PBVS, where perfect cross track error measurements are available.
Cross track error is then used to command desired heading as follows,

\[ \chi_d = \chi_f + \frac{\pi}{2} - \frac{\pi}{1 + e^{Ce_k s}} \]  

(B.1)

where \( \chi_d \) is desired bearing, \( \chi_f \) bearing angle of the feature, \( C_e \) cross track error and \( k_s \) the tuning parameter that controls rate of approach.

The third source of control represents the IBVS controllers and transition guidance solutions developed throughout this research. In total three controllers were developed that require varying levels of feedback. In the case of the STT IBVS controller presented in Chapter 3, only visual information is required, operating independent of the autopilot system. The FS controller of Chapter 4 requires state feedback and thus is connected to the sensor signals of the autopilot, these are then passed through a Kalman filter before providing feedback for control. The third design of Chapter 5 compensates for wind conditions and requires an estimate of mean wind and is provided to the IBVS controller before the addition of gust or turbulence.

B.1.3 Wind

Wind conditions are simulated in three components, mean wind, turbulence and gusts. Mean wind is set at the beginning of the simulation and remains constant for the duration of the test and does not include a vertical component. Turbulence is generated using a von Karman turbulence model included in the AeroSim Blockset that conforms to Military Specification MIL-F-8785C [222], and is a function of mean wind. Wind gusts are added to mean wind conditions in both the form of isolated (full 1-cosine function) and sustained gusts (two 1-cosine ramp functions allowing the disturbance to be held over a longer period), where forms for both are detailed in Section 5.2.2.

B.1.4 Simulated Imagery

Simulated imagery provides visual feedback for control, and utilises the world model of infrastructure to transform key turning points from an Earth fixed coordinate system (Latitude, Longitude, Altitude) to the camera sensor frame \((x_c, y_c, z_c)\) such that imagery as would be captured by a downward facing sensor
can be simulated. A detailed description of transformations used in this process is provided in the following section.

B.1.5 Sensor Models

In order to reflect real world conditions within the simulation environment, noise was added to each of the states that would be used by the FS IBVS controller in Chapters 4 and 5 (the STT IBVS controller of Chapter 3 does not require state information). Noise was introduced in the form of additive zero mean Gaussian noise, where variances were chosen based on trial and error and previous experience. Specifically, variances of 0.05 were chosen for velocity and angular measurements \((v, \phi, \Theta)\), 0.1 for rotational rates \((p, r)\), 0.01 for control surface deflections \((\delta_a, \delta_r)\) and 0.2 for Sensor Track Error, \(T_e\).

B.1.6 State Estimation

A Discrete Time Linear Kalman filter is included in Section 4.2.1 as part of the development of the FS IBVS controller in Chapter 4. Covariance matrices \(Q\) and \(R\) were selected as constants and chosen as follows,

\[
Q = \begin{bmatrix}
0.001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.001 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.001 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.01 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.01 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.001 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.001 & 0 \\
\end{bmatrix}
\]

\[
R = 10^4 \cdot I_{(8,8)}
\]

where weighting was placed on feature parameters to account for reduced measurement rates of 10 Hz compared to 100 Hz of the aircraft states.
B.2 Simulating Data Capture

In order to provide visual feedback for control, it was necessary to simulate imagery as would be captured by a sensor fixed to the UAV. The method for generating the data was derived from the concept of projecting key points within the simulation environment (i.e. power poles) from locations specified on the ground to their projected view in the image plane of an onboard sensor. These points could then be joined within the image plane to generate a simulated view of the feature, as illustrated in Figure B.3. The same concept could then be extended to simulate power lines, power poles, cross members and surrounding corridors, providing a more realistic view of data collection, as shown in Figure B.4.

![Figure B.3: Generation of image feature from projection of power poles.](image-url)
This technique does remove the catenary of the line, however would be considered negligible considering the perspective from which the UAV would generally observe the feature. The model does have the ability to account for variations in terrain, although is considered flat for the work presented.

\[ x_e = (R_N + h) \cos \phi \sin \lambda \]
\[ y_e = (R_N + h) \cos \phi \sin \lambda \]
\[ z_e = [R_N(1 - e^2) + h] \sin \phi \]

**Figure B.4:** Simulated view of feature as would be captured by downward facing sensor, including power lines, power poles, cross members and corridor.

**B.2.1 Transformations**

The following section details the coordinate transformations used in generating simulated imagery. The location of features would be assumed made available in Latitude, Longitude and Altitude (LLA), as would be provided by a GIS system, and transformed to the sensor frame fixed to the UAV.

**LLA to ECEF**

The following provides the transformation of coordinates expressed in geodetic latitude, longitude and altitude \((\phi, \lambda, h)\) to Earth Centred Earth Fixed (ECEF) coordinates \((x_e, y_e, z_e)\), as illustrated in Figure A.2.
where $R_N$ is the traverse radius of curvature and $e$ the ellipsoid eccentricity given by,

$$R_N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

where $a$ is the semi-major axis (equatorial radius) and $b$ the semi-minor axis (polar radius) of the reference ellipsoid, where the World Geodetic System 1984 (WGS84) used by GPS defines $a = 6 378 137$ m and $b = 6 356 752.3$ m.

**ECEF to NED**

The following provides the transformation of Earth Centred Earth Fixed (ECEF) coordinates $(x_e, y_e, z_e)$ to a local vertical North-East-Down (NED) Cartesian coordinate frame $(x_o, y_o, z_o)$ defined at $(\phi_o, \lambda_o, 0)$, as illustrated in Figure A.2;

$$
\begin{bmatrix}
x_o \\
y_o \\
z_o
\end{bmatrix} =
\begin{bmatrix}
- \sin \phi_o \cos \lambda_o & - \sin \phi_o \sin \lambda_o & \cos \phi_o \\
- \sin \lambda_o & \cos \lambda_o & 0 \\
- \cos \phi_o \cos \lambda_o & - \cos \phi_o \sin \lambda_o & - \sin \phi_o
\end{bmatrix}
\begin{bmatrix}
x_e - o_x \\
y_e - o_y \\
z_e - o_z
\end{bmatrix}
$$

where $(o_x, o_y, o_z)$ is the origin of the local system expressed in ECEF coordinates.

**Local Vertical to Body-Fixed**

The following provides the transformation from local vertical coordinates NED $(x_o, y_o, z_o)$ to the Body-Fixed frame of the aircraft $(x_b, y_b, z_b)$ as illustrated in Figure A.1;

$$
\begin{bmatrix}
x_b \\
y_b \\
z_b
\end{bmatrix} = {}^b R_o
\begin{bmatrix}
x_o \\
y_o \\
z_o
\end{bmatrix}
$$

where ${}^b R_o$ is the *Direction Cosine Matrix* that relates aircraft attitude with respect to the reference frame through Euler angles $(\Theta, \Phi, \Psi)$, given by;

$$
{}^b R_o = 
\begin{bmatrix}
\cos \Theta \cos \Psi & \cos \Theta \sin \Psi & - \sin \Theta \\
\sin \Phi \sin \Theta \cos \Psi - \cos \Phi \sin \Psi & \sin \Phi \sin \Theta \sin \Psi + \cos \Phi \cos \Psi & \sin \Phi \cos \Theta \\
\cos \Phi \sin \Theta \cos \Psi + \sin \Phi \sin \Psi & \cos \Phi \sin \Theta \sin \Psi - \sin \Phi \cos \Psi & \cos \Phi \cos \Theta
\end{bmatrix}
$$
where order of rotation is Yaw (Ψ), Pitch (Θ), Roll (Φ).

**Body-Fixed to Camera Frame**

The following provides the transformation of Body-Fixed coordinates \((x_b, y_b, z_b)\) to a Camera Frame \((x_c, y_c, z_c)\), positioned at \((c_{ox}, c_{oy}, c_{oz})\) and mounted at an azimuth of \(\phi_c\), tilted from vertical by \(\theta_c\), as illustrated in Figure B.5;

\[
\begin{bmatrix}
    x_c \\
    y_c \\
    z_c \\
\end{bmatrix} =
\begin{bmatrix}
    \cos \theta_c \cos \phi_c & \cos \phi_c \sin \lambda_o & \sin \theta_c \\
    -\sin \phi_c & \cos \phi_c & 0 \\
    \sin \theta_c \cos \phi_c & \sin \theta_c \sin \phi_c & \cos \theta_c \\
\end{bmatrix}
\begin{bmatrix}
    x_b - c_{ox} \\
    y_b - c_{oy} \\
    z_b - c_{oz} \\
\end{bmatrix}
\]

**Figure B.5**: Camera frame with respect to Body-Fixed frame.

Transformation from the camera frame to the image plane can then achieved through appropriate selection of a camera model as discussed in Appendix A.3.
Simulation Environment
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