Optimal Integration of GPS with Inertial Sensors: 
Modelling and Implementation

By

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Integration of GPS with Inertial Navigation Systems (INS) can provide reliable and complete positioning and geo-referencing parameters including position, velocity, and attitude of dynamic platforms for a variety of applications. This research focuses on four modelling and implementation issues for a GPS/INS integrated platform in order to optimise the overall integration performance:

a) Time synchronization
Having recognised that having a precise time synchronisation of measurements is fundamental in constructing a multi-sensor integration platform and is critical for achieving high data fusion performance, various time synchronisation scenarios and solutions have been investigated. A criterion for evaluating synchronisation accuracy and error impacts has been derived; an innovative time synchronisation solution has been proposed; an applicable data logging system has been implemented with off-the-shelf components and tested.

b) Noise suppression of INS raw measurements
Low cost INS sensors, especially MEMS INS, would normally exhibit much larger measurement noise than conventional INS sensors. A novel method of using vehicle dynamic information for de-noising raw INS sensor measurements has been proposed in this research. Since the vehicle dynamic model has the characteristic of a low pass filter, passing the raw INS sensor measurements through it effectively reduces the high frequency noise component.

c) Adaptive Kalman filtering
The present data fusion algorithms, which are mostly based on the Kalman filter, have the stringent requirement on precise a priori knowledge of the system model and noise properties. This research has investigated the utilization issues of online stochastic modelling algorithm, and then proposed a new adaptive process noise scaling algorithm which has shown remarkable capability in autonomously tuning the process noise covariance estimates to the optimal magnitude.
d) Integration of a low cost INS sensor with a standalone GPS receiver
To improve the performance where a standalone GPS receiver integrated with a MEMS INS, additional velocity aiding and a new integration structure has been adopted in this research. Field test shows that velocity determination accuracy could reach the centimetre level, and the errors of MEMS INS have been limited to such a level that it can generate stable attitude and heading references under low dynamic conditions.
ACKNOWLEDGEMENTS

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software package which has been used as the most important reference benchmark in this research work.

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<tr>
<td>A/D</td>
<td>Analogue-to-Digital convertor</td>
</tr>
<tr>
<td>b-frame</td>
<td>Body frame</td>
</tr>
<tr>
<td>C/A code</td>
<td>Coarse/Acquisition code</td>
</tr>
<tr>
<td>CDGPS</td>
<td>Carrier phase differential GPS</td>
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<tr>
<td>CDMA</td>
<td>Code-Division Multi-Access</td>
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<tr>
<td>CEP</td>
<td>Circular error probability</td>
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<tr>
<td>CORS</td>
<td>Continuously Operating Reference Station</td>
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<tr>
<td>DCM</td>
<td>Direction Cosine Matrix</td>
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<tr>
<td>DGPS</td>
<td>Differential GPS</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree Of Freedom</td>
</tr>
<tr>
<td>ECEF</td>
<td>Earth Centred Earth Fixed</td>
</tr>
<tr>
<td>e-frame</td>
<td>the Earth frame</td>
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<tr>
<td>EKF</td>
<td>Extended Kalman filter</td>
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<tr>
<td>FLL</td>
<td>Frequency lock loop</td>
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<tr>
<td>GAGAN</td>
<td>GPS Aided Geo Augmented Navigation</td>
</tr>
<tr>
<td>GNSS</td>
<td>Global Navigation Satellite Systems</td>
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<tr>
<td>GPS</td>
<td>Global Positioning System</td>
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<tr>
<td>I/O</td>
<td>Input/Output</td>
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<tr>
<td>IAG</td>
<td>International Association of Geodesy</td>
</tr>
<tr>
<td>i-frame</td>
<td>Inertial frame</td>
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<tr>
<td>IGS</td>
<td>International GNSS service</td>
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<tr>
<td>IMU</td>
<td>Inertial measurement unit</td>
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<tr>
<td>INS</td>
<td>Inertial Navigation System</td>
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<tr>
<td>ISA</td>
<td>Inertial sensor assembly</td>
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<tr>
<td>KF</td>
<td>Kalman filter</td>
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<tr>
<td>LOS</td>
<td>Line-Of-Sight</td>
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<tr>
<td>MEMS</td>
<td>Micro-Electro-Mechanical-Systems</td>
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<tr>
<td>NAVSTAR</td>
<td>NAVigation Satellite Timing And Ranging</td>
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<td>NDGPS</td>
<td>Nationwide Differential GPS System</td>
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<td>NED</td>
<td>North-East-Down</td>
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<td>n-frame</td>
<td>Navigation frame</td>
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<td>P code</td>
<td>Precise code</td>
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<td>PLL</td>
<td>Phase lock loop</td>
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<td>PPS</td>
<td>Precise Positioning Services</td>
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<td>Pseudorandom Noise</td>
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<td>Q matrix</td>
<td>covariance matrix of process noises in Kalman filter</td>
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<td>QZSS</td>
<td>Quasi-Zenith Satellite System</td>
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<td>R matrix</td>
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<td>RMS</td>
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<td>RTK</td>
<td>Real-Time Kinematic positioning</td>
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<td>SA</td>
<td>Selective Availability</td>
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<td>Simultaneous localization and mapping</td>
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<td>STD</td>
<td>Standard deviation</td>
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<tr>
<td>SVN</td>
<td>Space Vehicle Number</td>
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<tr>
<td>TEC</td>
<td>Total electron count</td>
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<tr>
<td>UAV</td>
<td>Unmanned Autonomous Vehicle</td>
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<tr>
<td>UTC</td>
<td>Coordinated Universal Time</td>
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<tr>
<td>WAAS</td>
<td>Wide Area Augmentation System</td>
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<tr>
<td>WGS84</td>
<td>World Geodetic System 1984</td>
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<td>ZUP</td>
<td>Zero velocity UPdate</td>
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Note: Variables used in each chapter are independently defined.
CHAPTER 1
INTRODUCTION

1.1. BACKGROUND

Over the past two decades, Global Navigation Satellite System (GNSS) have evolved to be such a revolutionary technology that it enables applications people had previously never dreamed of. It is such an enabling technology that nowadays countless applications are based on GNSS, from land surveying to car navigation, from guidance of unmanned autonomous vehicle to robotics control (Brown and Hwang, 1997; El-Sheimy, 2005; Farrell and Barth, 1998; Ford, 2004; GPS-Wing, 2008; Grejner-Brzezinska, 2004; Grejner-Brzezinska et al., 2006; Grewal et al., 2001; Hofmann-Wellenhof et al., 2001; Kremer, 2002; Rizos et al., 2005). Ubiquitous Positioning is increasingly becoming a part of people’s daily life (Meng et al., 2007; Retscher and Kealy, 2006).

Of such a fantastic technology, the Global Positioning System (GPS) from the USA is the pioneer, and is the only system that is currently fully operational (GPS-Wing, 2008; Rizos et al., 2005). Not far behind, GLONASS which is a Russian operated GNSS system, is quickly returning to full operational as well as new comers include the European Galileo system, the Chinese Compass system and other regional systems from countries such as India and Japan. Next generation GNSS technology is well under way, together with various GNSS augmentation systems. All are aiming to provide a more accurate, available and reliable positioning and timing services.

Together with the maturing of GPS technology, the integration of GPS and inertial navigation systems (INS) has attracted great interest from both industry and the academic community (Farrell and Barth, 1998; Grewal et al., 2001; Skaloud, 2002; Wang, 2000; Wendel and Trommer, 2004). Although GPS is being used where possible to replace conventional positioning and navigation technology due to its superior positioning performance, GPS alone is still not sufficient for providing a ubiquitous
spatial information solution (Grewal et al., 2001; Lee, 2002). This is due to some of the intrinsic weaknesses of GNSS systems. Since GPS positioning is relying on radio signals, it is highly sensitive to an attenuated signal environment, signal interference and jamming due to the weak transmission power of GPS signals. Signal blockage inside a building, under a bridge or inside a tunnel may stop GPS from functioning properly. Heavy canopy in the forest or a limited sky view in urban canyons may also severely degrade GPS performance. For example, heavy multi-path effects in city centres may cause some early GPS car navigation systems to deliver ridiculous information which make them totally unusable. In view of functionality, a standard GPS receiver can not provide critical attitude information which is required by most real time navigation applications. Although multiple antenna solutions may generate attitude measurements, its update rate and accuracy is relatively low for applications like direct geo-referencing in mobile photogrammetry, guidance and control of unmanned autonomous vehicles. To provide a more robust and reliable solution of position, velocity and attitude measurements with a high output bandwidth, integration of GPS and inertial navigation system is often the primary choice (Boeing, 1997; Farrell and Barth, 1998; GPS-Wing, 2008; Grewal et al., 2001).

At the same time, inertial navigation technology has also experienced a quick development phase in the last two or three decades since the strapdown technology became practical with powerful on-board computers (Titterton and Weston, 1997). A tactical grade INS which is an expensive device used mainly for high end military applications can now be available for only more than ten thousands dollars. Some low-grade individual accelerometers or gyroscopes can even cost as little as several dollars each if they are manufactured using micro machinery technology. Nowadays, low cost inertial sensors are being widely used in civil applications like automobile collision detection, antenna platform stabilisation, and hard disk vibration protection (Crossbow, 2008; GPS-Wing, 2008).

The integration of GPS and INS is a perfect match in order to deal with new application challenges. GPS provides long term positioning accuracy and stability, and its errors do not accumulate over time; but in the short term GPS may have relatively larger errors and the GPS measurements have a low sampling rate. In contrast, INS has a
comparatively high short-term accuracy and a high sampling rate. It is autonomous, self-contained and has no outages. But in the long term, its error may grow exponentially with respect to time and become unbounded due to the fact that its positioning and attitude solutions come from the integration of accelerations and angular rates measurements. GPS/INS integrated systems offer many advantages over a stand-alone GPS or INS system due to their complementary characters. INS aiding improves overall immunity to GPS outages, and reduces the GPS ambiguity search load in tightly coupled systems. INS can also provide a continuous attitude solution to the integrated system. GPS can bound and calibrate inertial sensor errors and hence low grade IMU can be used for high precision navigation applications.

The concept of merging GPS and INS together to get better accuracy, reliability and availability has been well accepted by most researchers and industry. Some successful products are already available in the market, for example, direct geo-referencing systems for mobile photogrammetry and gravimetry. However, to extend this conventionally high-end, cumbersome system to much wider civilian applications with more affordable configurations and better availability, reliability and mobility, is still a significant challenge in different technical strata, especially when the high dynamic, real-time and precise carrier phase positioning requirements are needed in many applications.

1.2 GPS OVERVIEW

1.2.1 GPS principles

GPS is a U.S. owned satellite navigation system capable of providing a highly accurate, continuous global navigation service independent of other positioning aids. GPS provides 24-hour, all-weather, worldwide coverage with position, velocity and timing information. The GPS system consists of three segments: 1) the space segment; 2) the control segment; 3) the user segment. The U.S. Air Force develops, maintains, and operates the space and control segments (GPS-Wing, 2008).
Chapter 1 Introduction

- The space segment is composed of more than 24 active NAVSTAR (NAVigation Satellite Timing And Ranging) satellites. The satellites forming the nominal constellation are distributed in six 55° orbital planes, with four satellites in each plane. The orbit period of each satellite is approximately 12 hours at an altitude of 20,183 km. This provides a GPS receiver with at least four satellites in view from any point on Earth, at any particular time. The GPS satellite signal identifies the satellite and provides the positioning, timing, ranging data, satellite status and the corrected ephemerides of the satellite to the users. The satellites can be identified either by the Space Vehicle Number (SVN) or the Pseudorandom Code Number (PRN).

![Figure 1.1 GPS satellite constellation (Crossbow, 2008; GPS-Wing, 2008)](image)

Figure 1.1 GPS satellite constellation (Crossbow, 2008; GPS-Wing, 2008)

- The control segment consists of worldwide monitor and control stations that maintain the satellites in their proper orbits through occasional command manoeuvres, and adjustments of the satellite clocks. It tracks the GPS satellite, uploads updated navigation data, and maintains the health status of the satellite constellation.

- The user segment consists of various GPS receiver equipment, which receive the signals from the GPS satellites and use the received information to calculate the user’s latitude, longitude, altitude and the GPS system time. A minimum of four satellites in view are needed to allow the receiver to compute a valid solution. The GPS employs the Earth Centred Earth Fixed (ECEF) World Geodetic System 1984 (WGS84) as the reference frame to which all GPS positioning and navigation
information is referred. GPS time was zero at Coordinated Universal Time (UTC) 0h 6-Jan-1980. Since it is not perturbed by leap seconds, GPS time is now ahead of UTC by 14 seconds.

GPS has such a dual-use nature that it provides services to civilian and military users at the same time. The Standard Positioning Services (SPS) is freely available to all civilian users, which has a Signal-in-Space performance (without consideration of contributions of ionosphere, troposphere, receiver, multipath, or other interferences) of 13 meters 95% horizontal error and 22 meters 95% vertical error (PNT, 2001). The Precise Positioning Services (PPS) is only available to U.S. and its allied arm forces as well as approved Government agencies.

In order to further explore the GPS positioning potential, a variety of GPS augmentation systems and techniques have been developed to meet specific requirements in terms of availability, accuracy, and integrity. These augmentation techniques include: 1) Nationwide Differential GPS System (NDGPS); 2) space based augmentation systems like the Wide Area Augmentation System (WAAS), OmniSTAR; 3) other regional based augmentations like Quasi-Zenith Satellite System (QZSS), and GPS Aided Geo Augmented Navigation (GAGAN). Modernization programs are underway to further improve GPS performance by the implementation of a second and third civil signal on GPS satellites.

1.2.2 GPS observables

Each GPS satellite transmits ranging code and navigation data using code-division multi-access (CDMA) technique with two radio frequencies denoted by L1 (1575.42 MHz) and L2 (1227.60 MHz). Three pseudorandom noise (PRN) codes are exclusively associated with each satellite and are modulated onto the L1 and L2 carriers along with the navigation message. The L1 frequency is modulated by the Coarse/Acquisition code (C/A code), Precise code (P code), and navigation message, see Equation (1.1) (Hofmann-Wellenhof et al., 2001). The L2 frequency is modulated by the P code and the navigation message, see Equation (1.2) (Hofmann-Wellenhof et al., 2001). The C/A code has a wave length of 293m with a chip rate of 1.023 MHz while the P code has a
wave length of 29.3m with a chip rate of 10.23MHz. When anti-spoofing is active, the P-code is encrypted to the Y-code using the W-code as the cryptographic key. Included in the navigation messages are GPS timing information, satellite ephemeris, clock correction and system status.

\[
s_{L_1}(t) = \sqrt{2P_I d(t)}c(t)\cos(\omega_1 t + \theta_1) + \sqrt{2P_Q d(t)}p(t)\sin(\omega_1 t + \theta_1)
\]

\[
s_{L_2}(t) = \sqrt{2P_Q d(t)}p(t)\sin(\omega_2 t + \theta_2)
\]

where \( s_{L_1}, s_{L_2} \) denote the signal wave forms of L1 and L2;
\( d \) denotes the modulation of the 50-bps navigation messages;
\( P_I, P_Q \) are the respective carrier powers for the in-phase and quadrature-phase carrier components;
\( c, p \) are the respective C/A code and P code wave forms;
\( \omega_1, \omega_2 \) are the L1 and L2 carrier frequencies in radians per second;
\( \theta_1, \theta_2 \) are initial phase shifts in radians;

The receiver observables that are generally used for navigation processing are the code phase (Pseudorange), carrier phase, and carrier Doppler frequency. More details will be given in the following sections.

1.2.2.1. Pseudorange

The range from a GPS receiver to a GPS satellite can be determined by measuring how long it takes for the electronic radio signals to propagate from the satellite to the receiver, and then multiplying that propagation time by the speed of light (Hofmann-Wellenhof et al., 2001).

\[
\rho = c(t_r - t_s)
\]

Where \( t_r \) is the time at which the signal is received;
\( t_s \) is the time at which the signal is transmitted;
\( c \) is the speed of light (2.99792458 \( \times \) 10\(^8\) m/s);
\( \rho \) represents the distance between the position of the satellite at epoch \( t_s \) and the position of the GPS receiver at epoch \( t_r \). It is worth mentioning that \( t_r \) is measured according to receiver clock time; while \( t_s \) is measured according to GPS satellite clock time.

Since the GPS satellite clocks and the GPS receiver clock are not physically synchronized and both of them may have errors when compared to the standard UTC time, the raw range measurements obtained may include the biases caused by the satellite clock error and the receiver clock error. Additionally, errors from other sources such as ionosphere and troposphere effects, multi-path and electronic noise, would also contribute to the total range measurement error. Thus, the raw measurement obtained in the above way is referred to as a pseudorange measurement. It is the most fundamental measurement obtained by a GPS receiver.

1.2.2.2 Carrier phase

Besides making code phase observations, most GPS receivers also support accurate tracking of the carrier onto which the code was modulated. At any epoch \( t \) in the GPS receiver time, the beat phase \( \phi_r \) between the reconstructed satellite carrier signal and the receiver reference carrier signal can be expressed as (Hofmann-Wellenhof et al., 2001):

![Geometrical interpretation of carrier phase](image)

Figure 1.2 Geometrical interpretation of carrier phase (Hofmann-Wellenhof et al., 2001)
\[\varphi_s = f \cdot t - f \cdot \left(\frac{\rho}{c}\right) - \varphi_{s0}\] (1.4)

\[\varphi_r = f \cdot t - \varphi_{r0}\] (1.5)

\[\varphi_{sr} = \varphi_s - \varphi_r = -f \cdot \left(\frac{\rho}{c}\right) - (\varphi_{s0} - \varphi_{r0})\] (1.6)

where \(\varphi_s\) is the phase of the reconstructed satellite carrier signal at epoch \(t\);
\(\varphi_r\) is the phase of the receiver reference carrier signal at epoch \(t\);
\(\varphi_{s0}\) is the initial phase of the reconstructed satellite carrier signal at epoch 0;
\(\varphi_{r0}\) is the initial phase of the reconstructed satellite carrier signal at epoch 0;
\(f\) is the nominal carrier phase frequency (of L1 or L2);
\(c\) is the speed of light \((2.99792458 \times 10^8 \text{ m/s})\);
\(\rho\) denotes the distance between the satellite and the GPS receiver;
\(t\) denotes the current epoch.

Suppose satellite carrier and receiver carrier initial phases are zero, we then get

\[\varphi_{sr} = \varphi_s - \varphi_r = -f \cdot \left(\frac{\rho}{c}\right)\] (1.7)

Since only the fractional part of the beat phase can be directly measured in the receiver, at any epoch \(t_0\), \(\varphi_{sr}\) can be split into an integer part \(N\) (full wave cycles) and a fractional part \(\theta(t_0)\) (phase angle within one cycle),

\[\varphi_{sr}(t_0) = -f \cdot \left(\frac{\rho(t_0)}{c}\right) = N + \theta(t_0)\] (1.8)

By continually tracking the change of the phase angle within one cycle, which is defined as \(\delta\theta\) here, the beat phase at epoch \(t\) can be expressed as

\[\varphi_{sr}(t) = -f \cdot \left(\frac{\rho(t)}{c}\right) = N + \theta(t_0) + \int_{t_0}^{t} \delta\theta dt\] (1.9)

Define GPS carrier phase observation as

\[\Phi(t) = \theta(t_0) + \int_{t_0}^{t} \delta\theta dt = \frac{\rho(t)}{\lambda} + N\] (1.10)

The initial integer number \(N\) is called the integer ambiguity, which needs to be resolved using statistical methods in order to exploit high accuracy positioning.
1.2.2.3 Doppler

If a wave transmitter and a receiver move with respect to each other, the apparent frequency of the received wave changes. For electromagnetic waves, the received frequency \( f_r \) can be computed according to Wieser (Wieser, 2007) as

\[
f_r = f_0 \left(1 + \frac{v_{\text{los}}}{v_0}\right)
\]

(1.11)

Where \( f_0 \) is the transmission frequency;

\( v_{\text{los}} \) is the relative line-of-sight (LOS) velocity between transmitter and receiver

where \( v_{\text{los}} > 0 \) if the transmitter and the receiver are approaching each other;

\( v_0 \) is the wave propagation speed.

The Doppler shift is then defined as:

\[
f_D = f_r - f_0 = f_0 \frac{v_{\text{los}}}{v_0}
\]

(1.12)

For GPS, the satellite can have a relative velocity of up to 800 m/s against a point fixed on the Earth’s surface and this can be even larger when the receiver is also moving (Wieser, 2007). All GPS receivers have to be able to track the Doppler shift of the radio signals for proper signal tracking. GPS Doppler measurements can either be obtained in the phase lock loop (PLL) or frequency lock loop (FLL). Both PLL and PLL circuits can provide a GPS receiver the capability to track the carrier frequency \( f_r \) of the received signals. The FLL provides a larger capture range of \( f_r \) without locking to phase. The PLL can keep a phase lock to \( f_r \) within a narrow capture range. More detailed introductions about the PLL and FLL have been given by Grewal et al. (Grewal et al., 2001).

Since the Doppler shift is proportional to the line-of-sight velocity between the GPS satellite and receiver, it can be transformed from unit of Hz into m/s by multiplying the wave length \( \lambda \) of the GPS radio signal:

\[
\lambda f_D = v_{\text{los}}
\]

(1.13)
1.2.3 GPS positioning

1.2.3.1 Error sources

- Satellite orbit error
Theoretically, the orbits of the GPS satellites in space follow Keplerian motion defined by six orbital parameters. In reality, there are many disturbing factors acting on the satellites, causing the temporal variations of the Keplerian elements (Hofmann-Wellenhof et al., 2001). They can be divided into two groups, namely those of gravitational origin which includes non-sphericity of the earth and tidal attraction, and those of non-gravitational origin which includes solar radiation pressure, air drag and so on.

Position and velocity vectors of GPS satellites can be determined from one of three data sets: almanac data, broadcast ephemerides, and precise ephemerides. Almanac data which is often packed in YUMA format provide less precise data for the purpose of satellite searching and task planning. In the broadcast ephemerides, several correction parameters are included based on the ground observations of the GPS control segment which delivers satellite orbits with 1 meter level RMS uncertainty. Higher accuracy up to 5 cm level is achievable with precise ephemerides as published by the Internal GNSS Service (IGS); however, there might be processing latencies from 15 minutes up to several hours or even days for these to become available. For single point positioning, the orbital error is highly correlated with final positioning error; while in the case of relative positioning, this error can be largely cancelled in the differencing processing.

- Satellite clock error
Even though there are four atomic clocks on board of each GPS satellite for maintaining precise time, the error of the satellite clock can accumulate as high as 1ms, causing significant range measurement errors. Due to this, the satellite clock bias, drift and drift rate are monitored continuously by the GPS control segment with respect to GPS standard time. Calibration parameters are estimated and uploaded to the satellites, then broadcast together with the other navigation messages.
Since GPS on-board clock error varies slowly, calibration can be carried out using a predefined polynomial with the coefficients transmitted in the navigation messages. After the correction has been applied, the residual satellite clock error is typically less than a few nanoseconds, or about 1m (RMS) in the range domain (Grewal et al., 2001).

- **Receiver clock error**

  Since most GPS receiver clocks use a quartz crystal oscillator with absolute accuracies in the 1 to 10ppm range, their long term stability is insufficient for positioning purposes. Fortunately, receiver clock error is common for all satellite measurements if the measurements are taken simultaneously. With more than four satellites in view, this error is normally included in the estimation as a variable along with the receiver coordinates.

- **Ionospheric Errors**

  The ionosphere, by definition, extends from approximately 50 to 1000km above the surface of the earth, and consists of gases that have been ionised by solar radiation (Grewal et al., 2001). Unlike the situation in free space, the propagation speed of GPS radio signals is changed when they penetrate through the ionosphere. The magnitude of this effect is directly proportional to the total electron count (TEC) along the path. The TEC varies with the ionospheric dynamics and with the changes of the propagation path of the satellite signals. At the zenith, the typical variation caused by signal travelling delay is about 5m to 15m during peak times. At a low elevation angle, this value can be as large as 50m. So in GPS positioning, a cut off mask from 5 degree to 15 degree is often set to restrict the use of measurements with lower elevation angles.

- **Tropospheric Errors**

  The troposphere refers to the layer of the lower part of the Earth’s atmosphere up to about 50km, which is normally treated as being composed of dry component (dry gases) and wet component (water vapour). When passing through this layer, GPS signals can be delayed from between 2.5m to 15m due to the nondispersive refraction effect.

  The delay caused by the dry gases which contributes 90% of total delay, is relatively stable so can be computed accurately from the pressure and temperature measured at the
receiver antenna and compensated for up to 95% to 98% (Grewal et al., 2001). The water component influence represents 10% of the total delay. It is far more variable because of the spatial and temporal change of the water vapour. In practice, a tropospheric model such as Hopfield or Saastamoinen model of the standard atmosphere at the antenna location would be used to estimate the combined zenith delay due to both dry and wet components. The total delay is calculated as the zenith delay multiplied by a factor that is a function of the satellite elevation angle (Grewal et al., 2001).

- **Multipath Errors**

GPS positioning relies on precise line-of-sight range measurement from each satellite. However, reflections of GPS signals from surfaces of objects near the antenna can cause the received signals being transmitted through paths other than the direct line-of-sight path. These non-line-of-sight signals would be superimposed on the true direct line-of-sight signals and cause distortion of the amplitude and phase of the true signal. This signal distortion consequently causes range measurement errors.

Multi-path errors are unlikely to be avoidable even with careful setups away from obvious reflections, and are difficult to model or compensate because of their arbitrary geometry distribution. Normally the signals received from low elevation satellites would be more susceptible to multipath influences than signals received from high elevation satellites. Long observation session would also smooth out intermittent multipath influences.

- **Residual noises**

Beside those error sources mentioned above, the remaining error sources normally contribute less in the total error budget. They are normally assumed to be uncorrelated and considered as white noises with zero mean over time in GPS positioning processing.
1.2.3.2 Single point positioning

In single-point positioning the coordinates of a GPS receiver at an unknown location are sought with respect to the Earth's reference frame by using the known positions of the GPS satellites being tracked. With four GPS satellites in view, the absolute position in three-dimensional space of one standalone receiver can be determined.

Considering most dominant errors existing in the pseudo-range measurement denoted as $P'_r$, its mathematical expression can be written as (Leick, 2004):

$$P'_r = \rho'_r + c\delta t'_s - c\delta t_r + I'_r + T'_r + M'_r + \varepsilon$$

(1.14)

where

$\rho'_r$ the geometric vacuum distance between the GPS satellite at epoch $s$ and the GPS receiver at epoch $r$. Strictly speaking, it’s a measurement between two points with different temporal epochs.

$\delta t'_s$ GPS satellite clock error;

$\delta t_r$ GPS receiver clock error;

$I'_r$ Ionospheric code delay;

$T'_r$ Tropospheric delay;

$M'_r$ Multipath delay;

$\varepsilon$ pseudorange measurement noise;

$c$ speed of light in the vacuum.

$$\rho'_r = \|\mathbf{x}' - \mathbf{x}_r\| = \sqrt{(x'_s - x_r)^2 + (y'_s - y_r)^2 + (z'_s - z_r)^2}$$

(1.15)

Where $\mathbf{x}'(x'_s, y'_s, z'_s)$ are the Earth-Centre-Earth-Fixed (ECEF) satellite coordinates;

$\mathbf{x}_r(, x_r, y_r, z_r)$ are the Earth-Centre-Earth-Fixed (ECEF) receiver coordinates.

After the linearization of Equation (1.15) and substituting the results into Equation (1.14), a first order linear equation matrix can be set up to solve the receiver position. In order to reduce the number of unknown variables in single point positioning, the error components in Equation (1.14) including satellite clock error, ionospheric delay,
tropospheric delay are corrected using priori models. The multipath errors are treated as pseudorange noise. The remaining variables are the receiver coordinates \( (x_r, y_r, z_r) \) and the receiver clock error \( \delta t_r \). That is why single point positioning needs at least four GPS satellites in view at the same time in order to generate a complete position solution.

The mathematical expression of carrier phase measurement is similar to Equation (1.14) (Leick, 2004),

\[
\lambda \Phi'_r = \rho'_r + \lambda N + c \delta t'_r - c \delta t_r + I'_r + T'_r + M'_r + \varepsilon
\]  

(1.16)

where the undefined symbols are

- \( \lambda \) the wave length of the carrier signal
- \( I'_r \) Ionospheric carrier phase delay; It is different form the value in the pseudorange expression.

Carrier phase measurements would only be used for smoothing pseudorange measurements in single point positioning. They can not be used independently for generating positioning solutions due to the difficulty of resolving the integer ambiguity in this mode.

**1.2.3.3 Differencing technique**

Detailed examination of Equations (1.14) and (1.16) reveals that most of the errors in the pseudorange and carrier phase measurements are transmission path dependant, so they are similar for receivers that are observing the same set of satellites and located in close proximity. In differential positioning, also known as relative positioning, the coordinates of a GPS receiver at an unknown location (called the rover station) are calculated with respect to a fixed GPS reference station (called the base station). The accuracy of differential positioning using two receivers can be significantly higher than single point positioning, especially when two receivers are close (less than 10km). This is because the largest error sources in single-point positioning are highly correlated for adjacent receivers and hence cancelled out in differential processing.
If the position of the base station can be determined to a high degree of accuracy using conventional surveying techniques, a high precision position of the rover station can be derived by only considering the relative distance between the rover and the base station. This differencing process is normally carried out in post processing mode in which both rover and base station data are brought together. If it is carried out in real-time mode, i.e. real-time kinematic (RTK), a real-time data link is required to connect two data sets. The difference between DGPS and RTK is that DGPS rover receives only error correction data like atmosphere modelling parameters from the base station; while in RTK, the rover needs to receive all base station raw measurements in real time, which is a significant amount of data.

One advantage of using carrier phase differential techniques is that carrier phase integer ambiguities can be solved, which enables positioning accuracy of up to a few centimetres. Many ambiguity fixing methods exists with the LAMBDA method being the most popular one (Teunissen and Tiberius, 1994).

1.3 INS OVERVIEW

1.3.1 INS principles

Inertial navigation technology has been used in an increasing number of civil applications for the guidance of aircraft, ships and land vehicles, as well as in other applications like robotic control, vibration detection etc.. The principles of this technology are based on the laws of mechanics formulated by Sir Isaac Newton, namely that a vehicle continues to move with the same velocity unless an external force is exerted and generates a proportional acceleration of the body.

Inertial navigation is a typical dead-reckoning system in that given an initial velocity and position, the following outputs of velocity and position are derived by performing successive mathematical integrations of the acceleration measurements. Normally, an inertial navigation system (INS) comprises three accelerometers and three gyroscopes (often referred as gyro) mounted in three orthogonal axes in order to carry out navigation tasks. This is due to the fact that three orthogonal acceleration measurements
have to be resolved in the chosen navigation reference frame, and the gravity force has to be extracted before they can be used for positioning.

Conventional inertial navigation systems use stable platform techniques. Although they are still common in high precision applications like submarine navigation, they are expensive and cumbersome, and gradually being phased out. Modern inertial navigation systems are more often in the strapdown form, with inertial sensors rigidly attached directly to the host vehicle. A more powerful computer is necessary to carry out more complex mathematical calculations when compared with that in a conventional system; in return the gains include cost and size reduction, and improved reliability. The majority of the currently implemented GPS/INS-integrated systems use strapdown INS.

Figure 1.3 shows the structure of a strapdown navigation system. An inertial sensor assembly (ISA) is a structure containing multiple inertial sensors (gyroscopes and/or accelerometers) which are orthogonal to one another. An inertial measurement unit (IMU) measures linear and angular motion in three dimensions without any external reference signals. The outputs of an IMU are incremental angles and velocities in the ISA frame. An inertial navigation system (INS) estimates a vehicle’s position, attitude, and velocity as a function of time in a specified navigation frame using the outputs of an IMU, a reference clock, and a model of the gravitational field (Curey et al., 2004).

Figure 1.3 Structure of a strapdown navigation system (Curey et al., 2004)
Table 1.1 Performance characteristics of inertial navigation systems

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Performance Parameters</th>
<th>Commercial MEMS</th>
<th>Tactical</th>
<th>Navigation I</th>
<th>Navigation II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gyroscope</td>
<td>Bias (deg/h)</td>
<td>&gt;100</td>
<td>1-10</td>
<td>0.1-1.0</td>
<td>0.002-0.01</td>
</tr>
<tr>
<td></td>
<td>Scale Factor (ppm)</td>
<td>&gt;1000</td>
<td>200-500</td>
<td>100-200</td>
<td>5-50</td>
</tr>
<tr>
<td></td>
<td>Noise Floor (deg/h/root-Hz)</td>
<td>0.2-0.5</td>
<td>0.05-0.2</td>
<td>0.002-0.005</td>
<td></td>
</tr>
<tr>
<td>Accelerometer</td>
<td>Bias (μg)</td>
<td>&gt;1000</td>
<td>200-500</td>
<td>50-100</td>
<td>5-10</td>
</tr>
<tr>
<td></td>
<td>Scale Factor (ppm)</td>
<td>&gt;1000</td>
<td>400-1000</td>
<td>100-200</td>
<td>10-20</td>
</tr>
<tr>
<td></td>
<td>Noise Floor (μg /h/root-Hz)</td>
<td>200-400</td>
<td>50</td>
<td>5-10</td>
<td></td>
</tr>
</tbody>
</table>

INS is often classified into several performance grades based on the error characteristics of the sensors. Table 1.1 shows the rough definition of the error ranges (Hewitson, 2006).

The advantage of using an inertial system is that it is self-contained, and can generate measurement updates very frequently. Its primary disadvantage is that its navigation accuracy drops quickly over time, which will be discussed in detail in the following sections.

1.3.2 Navigation calculation

Navigation calculation in an inertial navigation system involves conversions among several coordinate frames, such as the inertial frame (i-frame), the Earth frame (e-frame), the navigation frame (n-frame), and the body frame (b-frame) (Titterton and Weston, 1997):
• The inertial frame has its origin at the centre of the Earth and axes which are non-rotating with respect to the fixed stars. Its z-axis coincides with the Earth’s polar axis.

• The Earth frame has its origin at the centre of the Earth and axes with are fixed with respect to the Earth. Its z-axis is coincide with the Earth’s polar axis; its x-axis lies along the intersection of the plane of the Greenwich meridian with the Earth’s equatorial plane. The Earth frame can be represented in either Cartesian or geodetic form.

• The navigation frame (also called local frame) is based on the tangent plane to the Earth, with its origin at where the navigation system is located, and three axes aligned to the north direction, east direction and perpendicular down direction, respectively.

• The body frame is an orthogonal axis set with x, y and z axes coincide with the roll, pitch and yaw axes defined by inertial sensors assembly.

The acceleration measurements from accelerometers are actually expressed in the body frame; while angular rate measurements are indications of the angular change rates of the body frame axes with respect to the inertial frame. Since most navigation tasks are being carried out at the vicinity of the Earth surface, it is customary to express outputs of an inertial navigation system in the coordinates under the navigation frame.

The coordinate conversion of a point vector between two coordinate frames involves a translational transform and a rotational transform. For an inertial navigation system, translational transform is relatively easy when the translational displacement between two origins is clear. For rotational transform, it is necessary to know the orientation information of one frame with respect to the other. The attitude of an inertial navigation system is defined as the orientation of the body frame with respect to the navigation frame. It can be represented in several equivalent ways: 1) direction cosine matrix (DCM); 2) Euler angles; 3) quaternions.

The navigation equations in the navigation frame expressed by the Cartesian coordinate system can be described as follows:
\[ C^b_n = \int_0^t C^b_n \Omega^b_{ib} dt - \int_0^t \Omega^b_{ib} C^b_n dt \]  
(1.17)

\[ v^n = \int_0^t C^b_n f^b dt - \int_0^t \left[ 2 \omega^n_{en} + \omega^n_{en} \right] \times v^n dt + \int_0^t g^n dt \]  
(1.18)

\[ R^n = \int_0^t v^n dt \]  
(1.19)

where \( C^b_n \) is the transformation matrix from the body frame to the navigation frame;

\( \Omega^b_{ib} \) is a skew symmetric matrix formed by the angular rate vector sensed by gyroscopes, and expressed in the body frame;

\( \Omega^n_{in} \) is a skew symmetric matrix formed by the combined angular rate vector of the Earth and the orientation change of the navigation frame, and expressed in the body frame;

\( \omega^n_{en} \) is the Earth rotation rate vector expressed in the navigation frame;

\( \omega^n_{en} \) is the orientation change rate vector of the navigation frame expressed in the navigation frame;

\( f^b \) is the acceleration vector sensed by accelerometers, and expressed in the body frame;

\( g^n \) is the gravity vector expressed in the navigation frame;

\( v^n \) is the velocity vector in the navigation frame;

\( R^n \) is the position vector in the navigation frame.

1.3.3 INS errors

Inertial navigation system operating independently would exhibit large position errors that increase with time and are unbounded. This behaviour is largely due to the dead reckoning nature of an inertial navigation system, and the contamination of the inertial measurements by systematic and random errors, and imperfections in data processing such as initial alignment errors, gravity modelling error, linearization error and other numerical processing errors.
1.3.3.1 Sensor errors

Due to the dead-reckoning nature of an inertial navigation system, a very small bias error at the raw sensor output will eventually be integrated into a large positioning error. This places very high demands on sensor manufacturing accuracy, which is so high that often additional sensor calibration is necessary in order to reach the required performance.

Normally different type of sensors would exhibit different error characteristics due to their working mechanism and the method used to manufacture them. However, there are some common types of errors which represent the major error sources in an inertial navigation system, which will be briefly discussed.

- Fixed bias is the sensor output which is present even without external applied inputs of acceleration or rotation. It may be due to a variety of effects, including residual torques from flexible leads, temperature influences.
- Scale factor error is the error in the ratio between the change in the output signal and the change in the input, which includes bias, non-linearity, and asymmetry.
- Cross-coupling error is resulted from non-orthogonality of the sensor mounting axis.

Mathematically, inertial sensor errors can be expressed by the following two equations with Titterton and Weston (Titterton and Weston, 1997) convention, although specific components could vary depending on the sensor types.

For gyroscope:

\[
\omega_x = (1 + S_x)\omega_x + M_y \omega_y + M_z \omega_z + B_f + B_{gs} a_x + B_{gz} a_z + n_x \tag{1.20}
\]

where \( \omega_x, \omega_y, \omega_z \) measurements of the angular rate in three axes;

\( B_f \) g-insensitive bias;

\( S_x \) Scale factor error;

\( B_{gs}, B_{gz} \) g-sensitive bias coefficients;

\( M_y, M_z \) cross-coupling coefficients;
For accelerometer:

\[ a_x = (1 + S_x) a_x + M_y a_y + M_z a_z + B_f + n_x \]  \hspace{1cm} (1.21)

where \( a_x, a_y, a_z \) measurements of the angular rate in three axes;
\( B_f \) g-insensitive bias;
\( S_x \) Scale factor error;
\( M_y, M_z \) cross-coupling coefficients;
\( n_x \) random bias.

In the sense of error calibration and compensation, each of the errors described above is composed of a repeatable (fixed) component and a random component. The repeatable components can largely be corrected by manufacturing calibration in the factory, or be estimated using signal processing methods introduced later and compensated for. While the random components caused by the sensor instabilities will always be present, restricting the accuracy of inertial system performance.

In a practical application, often not all of those errors would be estimated and compensated on-line due to limited observability of the parameters. Only those with the largest impacts like the bias and scale factor errors would be modelled. The correction of remaining errors like cross coupling errors normally relies on manufacturer factory calibration data.

### 1.3.3.2 Alignment errors

Initial position, velocity and attitude measurements are required for an inertial navigation system to perform successive integrations of the acceleration measurements. Initial position and velocity is relatively easy to determine from external aiding sources like GPS, however finding a precise angular alignment often poses a certain difficulty. Initial angular alignment is the process of determining the orientation of the orthogonal axis of an inertial navigation system with respect to the external reference frame. It is
accomplished in two steps, namely coarse alignment and fine alignment. Coarse alignment is a process of finding the approximate values of the attitude angles, which again includes two processes called levelling and gyro compassing. In levelling, the roll and pitch angles are estimated using the accelerometer raw measurements; while in gyro compassing, the azimuth (yaw) angle is determined using gyroscopes raw measurements. Coarse alignment normally takes several seconds and its accuracy is limited by the accuracy of the raw sensor measurements. To tune the alignment further in the fine alignment stage, the alignment errors would be estimated using optimal estimation methods like a Kalman filter with observations of position and velocity information obtained from an external source. For example, whenever an inertial navigation system is informed that it has stopped moving, it can do a zero velocity update (ZUP), where it compares its output with the known fact that it is standing still on the Earth’s surface, and adjusts its navigation solution and estimates the error parameters accordingly. Fine alignment normally takes about 5 minutes to 15 minutes. After coarse alignment and fine alignment, any residual errors in the initial position, velocity and attitude remain for on line calibration.

Detailed examination of the navigation mechanization equation reveals that angular rate measurements play a more important role than accelerations in determining the overall accuracy of a navigation system. Further more, attitude errors have a much weaker observability than velocity and position errors in normal integration filter designs (Hong et al., 2005). So achieving high initial angular alignment is essential for improving the overall system performance.

1.3.3.3 Error models

Investigation of the error propagation behaviour in the navigation mechanism of an inertial navigation system is of great importance since with an accurate error propagation model, the impact of each error source can be investigated accordingly. When the navigation output has any discrepancy from the truth, the error sources can be traced back using mathematical methods like estimation techniques used in signal processing.
Two approaches used for deriving INS error propagation models are well known in the field, namely the perturbation approach and the psi-angle approach (Bar-Itzhack, 1988; Benson, 1975; Da et al., 1996). In the perturbation error model, the navigation equations are perturbed in the local-level north-pointing Cartesian coordinate system that corresponds to the true geographic location of the INS; while in the psi-angle error model, the navigation equations are perturbed in the local-level north-pointing coordinate system that corresponds to the geographic location calculated by the INS. Both models are equivalent and should yield identical results. The psi-angle error is the most widely adopted approach and has been implemented in many tightly coupled integration systems (Da, 1997; Da et al., 1996; Grejner-Brzezinska, 2004).

\[
\begin{align*}
\delta \dot{v} &= - (\omega_n + \omega_b) \times \delta v - \delta \psi \times f + \delta g + \nabla \\
\delta \dot{r} &= -\omega_{en} \times \delta r + \delta v \\
\delta \dot{\psi} &= -\omega_{en} \times \delta \psi + \epsilon 
\end{align*}
\]  

(1.22)

where \( \delta v, \delta r \) and \( \delta \psi \) are the velocity, position, and attitude error vectors respectively; 
\( \omega_b \) is the Earth rate vector; 
\( \omega_n \) is the angular rate vector of the true coordinate system with respect to the inertial frame; 
\( \omega_{en} \) is the angular rate vector of the true coordinate system with respect to the Earth; 
\( \nabla \) is the accelerometer error vector; 
\( \delta g \) is the error in the computed gravity vector; 
\( \epsilon \) is the gyro drift vector; 
\( f \) is the specific force vector.

1.3.4 MEMS INS

Micro-electro-Mechanical Systems (MEMS) is the integration of mechanical elements, sensors, actuators, and electronics on a common silicon die through micro fabrication technology (http://www.memsnet.org/mems/what-is.html). The technology allows both low cost manufacturing and the potential for even greater electronic/sensor integration.
As MEMS technology continues to mature and has demonstrated potential for use in many applications not previously achievable, there is a substantial effort being invested in the development of miniaturised inertial sensors which forms low cost MEMS based inertial navigation systems (Karnick et al., 2007; Mezentsev et al., 2007). At present, commercially available MEMS inertial sensors are at the level of 1-10 deg/sec of gyro bias, and 1% scale factor error and several milli-g acceleration bias. The latest efforts of the industry are targeting the development of a MEMS gyro with bias stability in the range of 0.1-1 deg/hour and a MEMS accelerometer with scale factor stability of 100 ppm (Mezentsev et al., 2007).

Compared with conventional navigational or tactical grade INS, low cost MEMS based inertial navigation systems are more susceptible to long-term and short-term errors. Low-cost MEMS INSs have shown to be usable in independent navigation only for tens of seconds without additional aiding. The biggest problem is the gyro drift rate that makes the gravity removal from the accelerometer measurements very difficult (Collin et al., 2001).

Integration with GPS would substantially mitigate MEMS INS drawbacks, and meet the performance, size, and cost goals raised by applications like navigation and control of unmanned autonomous vehicles, robotics control, automotive safety and car navigation systems and pedestrian navigation systems.

1.4 INTEGRATION OVERVIEW

GPS provides long term positioning accuracy and stability, and its errors do not accumulate. But in the short term GPS may have relatively larger errors (outliers). INS has comparatively high short-term accuracy and a high sampling rate. It is autonomous, self-contained and has no outages. But in the long term, its error grows exponentially and unbounded with respect to time due to the fact that its positioning and attitude solutions come from the integration of accelerations and angular rate measurements.
To bound the growth of inertial navigation errors, periodical calibration using an external information source is necessary, for example using external position or velocity fixes from GPS to reset position or velocity of an inertial navigation system from time to time whenever there is an external fix is the simplest way to bound error growth. However internal errors of inertial navigation systems are normally not observable to the external, so pure reset of position or velocity errors will not eliminate error growth in attitude estimation, or improve the inertial sensor performance. Hence more sophisticated solutions normally involve using data fusion technologies and inertial navigation system error propagation models in order that not only the position and velocity errors are corrected but also the attitude errors and inertial sensor errors like bias and scale factor drifts can be estimated and calibrated. Improvement on overall navigation accuracy and inertial sensor performance provides a less expensive alternative to the use of an unaided high grade inertial navigation system (Weston and Titterton, 2000).

Integration of GPS/INS offers many advantages over a stand-alone GPS or INS system due to their complementary characters.

- GPS can bound and calibrate inertial sensor errors and hence low grade inertial navigation systems can achieve higher performance;
- The inertial navigation system provides continuous attitude solutions to the integrated system;
- The inertial navigation system can work independently to provide navigation solutions while GPS signals are unavailable due to signal blockage;
- The integrated system can generate high-band-width outputs which are suitable for high dynamic navigation and control applications;
- In a tightly coupled configuration, aiding of inertial navigation system can be used to reduce GPS ambiguity searching load;
- In an ultra-tight integration, the GPS tracking band width can be reduced in order to improve immunity to jamming signals;
- The initial alignment of the inertial navigation system can be carried out much quicker, and partially on-line;
- Reliability of the integrated system is improved due to partial redundancy.
1.4.1 Integration strategy

When an extended Kalman filter is used in GPS/INS integrated systems, it is often implemented in a complementary form. The estimated states of the Kalman filter typically include corrections to the position, velocity, attitude and IMU sensor errors. The filtering outputs may be fed forward to the INS navigation outputs to generate integrated outputs, or fed back to the INS sensors to calibrate the sensor errors. The reason of using a complementary integration structure is that it provides a convenient reference trajectory for linearization of the non-linear dynamic and measurement relations. In addition, it reduces the tracking dynamics by estimating only the error components of the integrated system which change much slower than the direct positioning dynamics, and at the same time, maintains a high dynamic response by having a direct output channel (Brown and Hwang, 1997), see Figure 1.4. Practical implementation of a GPS/INS integrated system may have a variety of structures. In the literature, they are often roughly put into three categories, namely loose coupling (loosely coupled), tight coupling (tightly coupled), and ultra-tight coupling (ultra-tight coupled or deep coupled) (Swarna, 2006; Wang, 2007).

In the loose coupling mode, both GPS and INS are operating as independent subsystems. The position, velocity and attitude outputs from INS and position and velocity outputs from GPS are passed to the integration filter and blended there to generate final outputs (or corrections for the final outputs in complementary filtering form), as illustrated in Figure 1.4 (Cheng, 2005).

Figure 1.4 Loosely coupled integration
Loose coupling is normally considered as the simplest and quickest approach for GPS/INS integration. Since the two systems are operating independently, any fault to either subsystem can be isolated and located easily. It also provides a certain redundancy at system level which improves the overall robustness. The drawback of this approach is that GPS needs to keep tracking at least four satellites in order to generate a valid positioning solution. Otherwise the system will degrade to INS stand alone mode.

Tight coupling is a more complex approach, where raw pseudorange and carrier phase measurements from GPS, and raw accelerometer and gyroscope measurements are fed into a unified filter for data fusion, generating state estimates for navigation correction and INS error correction. In tight coupling, the GPS and INS are not operated as independent systems anymore, instead the positioning and navigation solution would only be generated at the central data fusion filter. Since the central filter needs to handle raw pseudorange and carrier phase processing and INS navigation calculation, the algorithms are much involved with each other and become very complicated. However, by integrating GPS and INS at the raw measurement level, tight coupling provides the benefit of better performance when the number of visible satellites drops below four. INS data can also be used for fixing GPS carrier phase integer ambiguity, and detecting cycle slips.

Ultra-tight coupling is an integration approach at an even lower level (refers to early stage of signal acquisition and processing). System dynamics sensed by the INS are fed into GPS signal tracking loops to enhance its tracking performance. Its main advantages are on robustness and anti-jamming due to the reduced tracking bandwidth (Swarna, 2006).

1.4.2 Data filtering

In a GPS/INS integrated system, the central component is an error filtering mechanism which provides estimates of the inertial navigation system errors based on a comparison of the measurement differences.
1.4.2.1 Kalman filter

Considering a multivariable linear discrete system:

\[
\begin{align*}
    x_k &= \Phi_{k-1}x_{k-1} + w_{k-1} \\
    z_k &= H_x x_k + v_k
\end{align*}
\]

(1.23) (1.24)

where \( x_k \) is \((n \times 1)\) process state vector at time \( t_k \);
\( \Phi_k \) is \((n \times n)\) state transition matrix;
\( z_k \) is \((r \times 1)\) observation vector at time \( t_k \);
\( H_x \) is \((r \times n)\) observation matrix;
\( w_k \) and \( v_k \) are uncorrelated white Gaussian noise sequences with means and covariances:

\[
\begin{align*}
    E\{w_k\} = E\{v_k\} &= 0 \\
    E\{w_kv_k^T\} &= 0 \\
    E\{w_kw_k^T\} &= \begin{cases} Q_i & i = k \\ 0 & i \neq k \end{cases} \\
    E\{v_kv_k^T\} &= \begin{cases} R_i & i = k \\ 0 & i \neq k \end{cases}
\end{align*}
\]

(1.25)

where \( E\{\cdot\} \) denotes the expectation function.

\( Q_k \) and \( R_k \) are the covariance matrix of process noise and observation errors, respectively.

The KF state prediction and state covariance prediction in time update procedure are

\[
\begin{align*}
    \hat{x}_k^- &= \Phi_{k-1}\hat{x}_{k-1} \\
    \hat{P}_k^- &= \Phi_{k-1}\hat{P}_{k-1}\Phi_{k-1}^T + Q_{k-1}
\end{align*}
\]

(1.26)
where \( \hat{x}_k \) denotes the KF estimated state vector; 
\( \hat{x}_i \) the predicted state vector;
\( \hat{P}_k \) the estimated state covariance matrix;
\( \hat{P}_k^{-} \) the predicted state covariance matrix.

The Kalman measurement update algorithms are:

\[
\begin{align*}
K_k &= \hat{P}_k^{-} H_k^T (H_k \hat{P}_k^{-} H_k^T + R_k)^{-1} \\
\hat{x}_k &= \hat{x}_k^{-} + K_k (z_k - H_k \hat{x}_k^{-}) \\
\hat{P}_k &= (I - K_k H_k) \hat{P}_k^{-}
\end{align*}
\] (1.27)

where \( K_k \) is the Kalman gain, which defines the updating weight between new measurements and predictions from the system dynamic model.

The innovation sequence is defined as:

\[
d_k = z_k - H_k \hat{x}_k^{-}
\] (1.28)

and the residual sequence as:

\[
\varepsilon_k = z_k - H_k \hat{x}_k
\] (1.29)

The conventional Kalman filter requires linear state space models where the system dynamics are actually expressed in a set of first order linear differential equations, and the measurement matrixes are a set of first order linear equations. When any one of them becomes non-linear, either the linearized Kalman filter or the extended Kalman filter (EKF) should be used. In the linearized Kalman filter, the nonlinear system models (either dynamics matrix or observation matrix) is linearized using a first order Taylor approximation about some nominal trajectory in state space that does not depend on the measurement data. While in the extended Kalman filter, the nonlinear system models is linearized about a trajectory that is continually updated with the state estimates generated from the Kalman filter measurement updates.
For a generic multivariable nonlinear discrete system,

\[ x_k = f(x_{k-1}, w_{k-1}) \]  
\[ z_k = h(x_k, v_k) \]  

Linearization of the system models can be expressed as

\[ \Phi_{k-1} = \left. \frac{\partial f(x_{k-1}, v_{k-1})}{\partial x_{k-1}} \right|_{x=x_{k-1}, v=x_{k-1}} \]  
\[ H_k = \left. \frac{\partial h(x_k, v_k)}{\partial x} \right|_{x=x_k, v=v_k} \]

Where \( f(\cdot), h(\cdot) \) denote arbitrary functions.

After linearization, the time update and measurement update of the EKF have the same procedure as for the linear Kalman filter. EKF is currently widely adopted in the implementation of tightly coupled GPS/INS integrated systems in order to handle the nonlinearity exhibited in the INS error models.

### 1.4.2.2 System and stochastic model

In this research, an INS phi-angle error model (Bar-Itzhack, 1977; Bar-Itzhack, 1988; Da et al., 1996; Grejner-Brzezinska, 2004) has been implemented as the system dynamic model for an extended Kalman filter, with additional state augmentation of INS errors, gravity modelling errors, and lever arm errors. A twenty four state extended Kalman filter (Da et al., 1996; Grejner-Brzezinska et al., 2005) is used for data fusion and error estimation, which includes nine navigation solution errors of three dimensional position, velocity and attitude, six accelerometer error modelling parameters (bias and scale factors for each axis), three gyro drifts, three gravity uncertainty errors, and three lever arm errors. Please refer to Equation (1.34) for details.
\[ \mathbf{x}_{\text{Nav}} = [\delta r_N, \delta r_E, \delta r_D, \delta v_N, \delta v_E, \delta v_D, \delta \psi_N, \delta \psi_E, \delta \psi_D]^{T} \]
\[ \mathbf{x}_{\text{Acc}} = [\nabla_{bx}, \nabla_{by}, \nabla_{bz}, \nabla_{fx}, \nabla_{fy}, \nabla_{fz}]^{T} \]
\[ \mathbf{x}_{\text{Gyro}} = [\varepsilon_{bx}, \varepsilon_{by}, \varepsilon_{bz}]^{T} \]
\[ \mathbf{x}_{\text{Grav}} = [\delta g_N, \delta g_E, \delta g_D]^{T} \]
\[ \mathbf{x}_{\text{Ant}} = [\delta L_{bx}, \delta L_{by}, \delta L_{bz}]^{T} \]

An exact expression of the dynamic matrix of the system model has the following form,
\[
\begin{bmatrix}
\dot{x}_{\text{Nav}} \\
\dot{x}_{\text{Acc}} \\
\dot{x}_{\text{Gyro}} \\
\dot{x}_{\text{Grav}} \\
\dot{x}_{\text{Ant}}
\end{bmatrix} =
\begin{bmatrix}
F_{11}(9 \times 9) & F_{12}(9 \times 6) & F_{13}(9 \times 3) & F_{14}(9 \times 3) & 0 \\
0 & F_{22}(6 \times 6) & 0 & 0 & 0 \\
0 & 0 & F_{33}(3 \times 3) & 0 & 0 \\
0 & 0 & 0 & F_{44}(3 \times 3) & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_{\text{Nav}} \\
x_{\text{Acc}} \\
x_{\text{Gyro}} \\
x_{\text{Grav}} \\
x_{\text{Ant}}
\end{bmatrix}
\begin{bmatrix}
w_{\text{Nav}} \\
w_{\text{Acc}} \\
w_{\text{Gyro}} \\
w_{\text{Grav}} \\
w_{\text{Ant}}
\end{bmatrix}
\] (1.35)

Where sub-matrixes and vectors could be further expanded as
\[
F_{11} =
\begin{bmatrix}
0 & -\dot{\lambda} sL & \dot{L} & 1 & 0 & 0 & 0 & 0 & 0 \\
-\dot{\lambda} sL & 0 & \dot{\lambda} cL & 0 & 1 & 0 & 0 & 0 & 0 \\
-\dot{L} & -\dot{\lambda} cL & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
-\frac{g}{R_e} & 0 & 0 & 0 & -(2\Omega + \dot{\lambda})sL & \dot{L} & 0 & -f_D & f_E \\
0 & -\frac{g}{R_e} & 0 & (2\Omega + \dot{\lambda})sL & 0 & (2\Omega + \dot{\lambda})cL & f_D & 0 & -f_N \\
0 & 0 & \frac{2g}{R_e} & -\dot{L} & -2\Omega + \dot{\lambda}cL & 0 & -f_E & f_N & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -(\Omega + \dot{\lambda})sL & \dot{L} \\
0 & 0 & 0 & 0 & 0 & (\Omega + \dot{\lambda})sL & 0 & (\Omega + \dot{\lambda})cL & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -(\Omega + \dot{\lambda})sL & \dot{L} & 0
\end{bmatrix}
\] (1.36)

where \( sL = \sin(L), \quad cL = \cos(L) \)

where \( \lambda, L \) are the local longitude and latitude;
\( g \) is the nominal gravity;
\( R_e \) is the Earth radius;
\( \Omega \) is the Earth rate
\( (f_N, f_E, f_D) \) is the specific force under navigation frame;
\[ F_{12} = \begin{bmatrix} 0(3 \times 3) & 0(3 \times 3) \\ \tilde{C}_b^n & \tilde{C}_b^n \\ 0(3 \times 3) & 0(3 \times 3) \end{bmatrix} \] (1.37)

\[ F_{13} = \begin{bmatrix} 0(3 \times 3) \\ 0(3 \times 3) \\ -C^n_b \end{bmatrix} \] (1.38)

Where \( C^n_b \) is the transformation matrix from body frame to navigation frame; \((f_x, f_y, f_z)\) is the specific force vector in body frame;

\[ F_{14} = \begin{bmatrix} 0(3 \times 3) \\ \mathbf{I}(3 \times 3) \\ 0(3 \times 3) \end{bmatrix} \] (1.39)

\[ F_{22} = \text{diag}[0, 0, 0, 0, 0, 0] \] (1.40)

\[ F_{33} = \text{diag}[0, 0, 0] \] (1.41)

\[ F_{44} = \text{diag}[-\tau_{gN}, -\tau_{gE}, -\tau_{gD}] \] (1.42)

Where \((\tau_{gN}, \tau_{gE}, \tau_{gD})\) is the time constant of the Markov process of the gravity model;

For stochastic modelling, all the process noises \([w_{\text{pos}}^T, w_{\text{av}}^T, w_{\text{aop}}^T]\) in the position, velocity, attitude and inertial sensor error states are modelled as zero-mean Gaussian white noises. The process noise \(w_{\text{Grav}}\) of gravity states is modelled as a Gauss-Markov process with time constants \((\tau_{gN}, \tau_{gE}, \tau_{gD})\). The antenna lever arm errors are modelled as random constants.
1.5 MOTIVATIONS AND RESEARCH OBJECTIVES

The concept of merging GPS/INS together to get improved accuracy, reliability and availability has arisen from the very early stage of GPS availability. After two decades of development, a lot of insight has been gained into the integration mechanisms. Besides abundant papers in research literature, several universities have developed prototype systems, such as VISAT™ (El-Sheimy, 1996), AIMS™ (Grejner-Brzezinska, 1998), HeliMap™ (Skaloud et al., 2005). Commercial systems like Position and Orientation Systems (POST™) from Applanix Corp. have been successfully used in a variety of applications such as aerial survey and mapping, remote sensing, road profiling, GIS data acquisition, and hydrographic surveying.

Nowadays, with the provided advantages that a GPS/INS integrated system can provide, both industry and research community are turning to the developments of much wider applications including car navigation, unmanned autonomous vehicle control, robotics control, and personal navigation. People believe such GPS/INS integrated systems could eventually be embedded into PDAs or mobile phones, together with other sensors to provide ubiquitous positioning services in daily life. However, extending this conventionally high-end, cumbersome system to much wider civilian applications still contains many research challenges in different technical strata. Some new demands versus current limitations are identified as below.

1) Affordability
   Building a direct geo-referencing system as mentioned above would easily cost more than AU$200,000. A dual frequency GPS receiver alone would cost about AU$20,000. Such costs are often not affordable for small or personal civilian applications. Providing an acceptable performance for most real time navigation and control tasks with affordable sensors, say standalone GPS receiver and MEMS INS, has exhibited many technical challenges. By employing more sophisticated software algorithms, for example raw measurement de-noising, adaptive or intelligent data filtering, or innovative data fusion architectures, people hope the affordability problem could be partially solved with more powerful on-board modelling and computation capacities.
2) Usability
Using current GPS/INS systems would require the users to have specialised knowledge and follow certain procedures including calibration, initial alignment or even tuning of Kalman filter. While for future applications, the integrated system should be able to work autonomously, and have the capability to adapt to different application environment.

3) Reliability
Some early versions of GPS car navigation systems would give ridiculous positioning information when operating in the urban canyon environment. This may happen due to limited sky view or multi-path influences. To mitigate the GPS shortages under harsh application environments, integration with INS is one of the most promising solutions.

To address the above challenges, this research has been focused on the following objectives:

a) Time synchronization issue
In building a compact GPS/INS integrated platform, time synchronisation of measurement data from GPS and INS sensors is a fundamental requirement. It has been recognised in the field of GPS/INS integration that having precise time synchronisation is critical for achieving high data fusion performance. Although there are many discussions in the literature (Lee et al., 2002; Li, 2004), a theoretical evaluation of the required time synchronisation accuracy for different GPS/INS applications is still missing. This research aims to provide a criterion for evaluating synchronisation accuracy requirements, and a generic and easy-to-implement time synchronization method.

b) Noise suppression of INS raw measurements
Since the INS navigation errors are exponentially proportional to the INS sensor noise level, reducing the noise level of the INS raw measurement is essential for reaching higher integration performance. Due to the difficulty of precisely modeling
these high frequency noises, de-noising or pre-filtering techniques are often employed to suppress them before they can contaminate the filtering process (El-Rabbany and El-Diasty, 2004; El-Sheimy et al., 2004; Grejner-Brzezinska et al., 2005; Nassar and EL-Sheimy, 2005; Nassar and El-Sheimy, 2006). One possible drawback of existing conventional filtering techniques is that there is no rigorous criterion to evaluate the cut-off frequency or the wavelet de-noising levels. Users have to be very cautious to prevent an over-smoothing effect (Grejner-Brzezinska et al., 2005). This research aims to investigate the plausibility of using vehicle dynamics in the pre-filtering (de-noising) process in order to avoid arbitrary determination of cut-off frequency.

c) Adaptive Kalman filtering
The present data fusion algorithms, which are mostly based on Kalman filtering (KF), have several limitations. One of those limitations is the stringent requirement on precise a priori knowledge of the system models and noise properties. Uncertainty in the covariance parameters of the process noise (Q) and the observation errors (R) may significantly degrade the filtering performance. The conventional way of determining Q and R relies on intensive analysis of empirical data. However, the noise levels may change in different applications. Over the past few decades adaptive KF algorithms have been intensively investigated with a view to reducing the influence of the Q and R definition errors. The covariance matching method has been shown to be one of the most promising techniques. However, some issues concerning the optimal window size for covariance estimation and the interaction between process noise estimation and observation noise estimation, and its stability in different application environment still need further investigation.

d) Integration of a low cost INS sensor with a standalone GPS receiver
As mentioned above, the applications of GPS/INS integrated systems are gradually turning from the current post processing mode to an on-line processing mode. Accuracy is not the only benchmark that users are looking for, but also real-time delivery ability, for example in UAVs and some emergency services. Also a standalone working mode is more favourable for lower end users in terms of cost reduction, and flexibility. To that end, this research aims to investigate an optimal
integration strategy for the scenario where a standalone GPS receiver is integrated with a MEMS INS.

1.6 CONTRIBUTIONS

The contributions of this research can be summarised as follows:

- A criterion for evaluating synchronisation accuracy and error impacts has been derived theoretically. In the literature, the requirements on synchronization accuracy in different implementations for different applications were normally estimated based on evaluating empirical data.

- An innovative time synchronisation solution has been proposed with a detailed block diagram showing the possible hardware implementation. The results can be directly used for implementing a data logger, or constructing an integration board.

- A novel method of using vehicle dynamic information for de-noising raw INS sensor measurements has been proposed.

- An online stochastic modelling algorithm has been investigated, regarding its parameter estimation stability, convergence, optimal window size, and the interaction between Q and R estimations, which provides better understanding of the benefits and limitations of this adaptive filtering approach.

- A new adaptive process noise scaling algorithm has been proposed, which has shown remarkable capability in autonomously tuning the process noise covariance estimates to the optimal magnitude.

- A new strategy for integration of a standalone GPS receiver with a low cost inertial navigation system has been proposed. This new approach solves the difficulty of precise calibration of INS errors in such a scenario, and enables MEMS INS to generate stable attitude and heading solutions under low dynamic conditions.
Chapter 1

Introduction

- A new algorithm for calculating GPS delta-distance of each sampling interval has been derived.

During this research period, there are 3 referred papers being published in journals, 6 referred papers and 3 more abstract-referred papers being published in conference proceedings (one paper has won the prize in the ION2007 student paper competition). Please refer to the author’s publications during the Ph.D studies.

1.7 THESIS OUTLINE

This thesis consists of six chapters. The contents of each chapter are outlined as follows.

Chapter 1 gives a general overview of GPS, INS principles, basic formulas and error source analysis. The INS error propagation algorithm is introduced, which forms the foundation for understanding the data filtering process. Then the Kalman filter algorithms and detailed implementation are given.

Chapter 2 investigates time synchronization issue in implementing multiple-sensor integration. Firstly various time synchronisation scenarios and solutions have been reviewed. Then time synchronization error propagation mechanisms have been derived. Based on the analysis, an innovative time synchronisation solution has been proposed and test results have been presented. The main content of this chapter has been published on the ETRI journal with paper title “Time synchronization error and calibration in integrated GPS/INS systems”.

Chapter 3 investigates the de-noising algorithms for INS raw measurements handling. After a review of existing methods, a working mechanism of a vehicle dynamic based solution is introduced. Implementation and test results are then presented. The main content of this chapter has been published in the proceedings of IGNSS2007 conference with paper title “Vehicle Dynamics Based De-Noising for GPS/INS Integration”.

Chapter 4 concentrates on adaptive Kalman filtering algorithms. In the first part, the utilization issues of online stochastic modelling algorithm are investigated. Then a new
adaptive process noise scaling algorithm is proposed. Results from field test data have been presented. The main content of this chapter has been published in the Journal of Navigation (the Royal Navigation Institute) with paper tile “Improving covariance based adaptive estimation for GPS/INS integration”.

Chapter 5 investigates a new strategy for integration of a standalone GPS receiver with a low cost inertial navigation system. With the context of low cost and MEMS INS sensors, the integration challenges are first reviewed. Then a two step aiding architecture is proposed and new delta-distance algorithm is derived. Then implementation and test results are presented. The main content of this chapter has been published in the ION 2007 conference with paper title “Integration of MEMS INS with GPS Carrier Phase Derived Velocity: A new approach”, which has won the ION best student paper prize.

Chapter 6 summarises the research findings, draws conclusions, and makes recommendations for future investigations.
CHAPTER 2
A METHOD FOR TIME SYNCHRONIZATION OF SENSORS

2.1 INTRODUCTION

Time synchronisation between GPS and inertial measurement unit (IMU) measurements is a common concern when implementing integrated GPS/INS systems. Since the GPS receiver and INS unit are two separated (self-contained) subsystems, the clock difference and data transmission latency can cause data alignment discrepancies during the data fusion stage. This results in the problem of time tagging bias, asynchronous sampling instants and different sampling rates between GPS and INS. Such alignment discrepancy may render the data fusion suboptimal. In many applications, the time synchronisation of additional sensors, such as barometer, odometer, and imaging sensor, might be necessary as well.

The time synchronisation issue has been extensively reported in the research literature and is considered to be one of the critical factors in achieving high integration accuracy. The report on mobile multi-sensor systems by the International Association of Geodesy (IAG) working group (El-Sheimy, 1999) acknowledges its importance. The proposal for IEEE inertial systems standard (Curey et al., 2004) suggests that the synchronisation of the INS internal clock to an external time reference like the GPS clock is an important issue to be addressed.

Early studies of time synchronization issues include Bar-Itzhack’s observation (Bar-Itzhack, 1977) of enigma bias during INS transfer alignment. (Knight, 1996) describes an accurate tagging of INS raw measurements with GPS time as probably the single most critical element of successful GPS/INS tight coupling. (Grejner-Brzezinska, 2004) shows that time synchronization is a factor that is crucial for achieving high accuracy positioning based on multi-sensor integration. (Li, 2004) has successfully designed a cost-effective experimental device to mitigate INS data transmission uncertainty.
Due to its critical role in GPS/INS integration, a further comprehensive and systematic study of time synchronization is needed to identify working mechanisms and to develop methods to mitigate time synchronization errors. In this chapter a detailed analysis of the limitations and advantages of different time synchronisation scenarios and existing solutions is presented. A criterion for evaluating synchronization accuracy requirements is developed based on the comparison of the Kalman filter innovation series and the platform dynamics. An innovative time synchronisation solution using a counter and two latching registers is proposed. The proposed solution is verified using a test system implemented with off-the-shelf components. Without an INS hard-wired timing signal, as reported in the test results, a synchronisation accuracy of 2 milliseconds was achieved when the test system was used to synchronise a low-grade INS sensor which is manufactured with Micro-Electro-Mechanical Systems (MEMS) technology, and has only a RS-232 data output interface. To simplify the theoretical analysis, this thesis has assumed that the INS sensor errors and time synchronization errors are uncorrelated from each other. This should be a reasonable simplification when we considering the nature of these two error sources.

2.2 SYNCHRONIZATION SCENARIOS

To demonstrate the time latencies in the GPS/INS integration context, a typical loosely coupled model is illustrated in Figure 2.1.

![Figure 2.1 Clock bias and transmission latency of loosely coupled mode](image-url)
A typical loosely coupled GPS/INS integration model is comprised of three functional subsystems, i.e. GPS receiver, INS, and integration platform. All of the three subsystems need to have internal clocks for their proper operations. The GPS receiver, due to the nature of its design, has the possibility to link its internal clock time to GPS Time, which is subsequently linked to UTC Time. Although there is a time difference between UTC Time and GPS Time (leap seconds), it is not relevant to the current issue since the time synchronization accuracy refers to the relative time difference between the GPS receiver and the INS. So the satellite time can be considered as a real time reference. Not every GPS receiver synchronizes its internal clock to satellite time all the time. It depends on individual receiver design. Technically speaking, a GPS receiver can synchronize its internal clock to satellite time (GPS Time) to within 50ns accuracy.

In contrast, INS works rather independently, and has no link to real time references but is operating in a free-run mode. This feature fundamentally creates the time synchronization problem for GPS/INS integration, because the data measured by the INS cannot match perfectly to GPS data by just using time tags as would be possible in other instrumentation systems. The only way the integration algorithm can do this is to probably align GPS and INS data according to the instant when they are processed by the integration platform.

Transmission and processing latency add to time synchronization ambiguity. Since in most cases GPS and INS data are transmitted using a serial communication link like RS-232(EIA-232), RS-422(EIA-422), time delay is unavoidable especially when a buffering technique is used and/or the communication load is heavy. Data losses may happen due to overflow when transmission load is heavy, data distortion caused by ambient interferences, and so on. Then the whole correspondence relationship of the data is disrupted.

Furthermore, running an integration algorithm like a Kalman filter is computationally intensive. Data arriving at the processing platform is usually put into a queue waiting for unpacking and error checking, before they are properly time tagged by the processing platform using the internal clock time.
2.3 ERROR ANALYSIS

2.3.1 Error propagation

The growth of errors in the horizontal channels of a Schuler-tuned INS is bounded by the Schuler tuning effects, while the errors in the vertical channels tend to grow exponentially with time. To deal with the large INS navigation errors and modelling non-linearities, the actual implementation of the Kalman filter in GPS/INS integration is often in the complementary form (Brown and Hwang, 1997). In such configurations, the observation vector in the Kalman filter is expressed by Equation (1.24) and is the difference between the GPS measurements and INS measurements. When expressed in the continuous time domain, the observation vector $z(t)$ is:

$$z(t) = r_{GPS}(t) - r_{INS}(t)$$

$$= r_{GPS}(t) - \left( r_{INS}(0) + \int_0^t v_{INS}(0)dt + \int_0^t \int_0^t a(t)dt \right)$$

(2.1)

Where $r_{GPS}(t)$ is the GPS position measurements, $r_{INS}(0), v_{INS}(0)$ are the INS initial position, velocity and $a(t)$ is the acceleration measured by the INS sensors in the proper coordinate frame. Assuming that the INS initial position and velocity is known, if there exists time synchronization errors in the INS measurements, the observation vector is changed to $\tilde{z}(t)$:

$$\tilde{z}(t) = r_{GPS}(t) - \left( r_{INS}(0) + \int_0^t v_{INS}(0)d\tau + \int_0^t \int_0^t a(\tau + \Delta t)d\tau d\sigma \right)$$

(2.2)

Where $\Delta t$ denotes the INS time synchronisation error. It can either be positive or negative and can be treated as a constant in the following discussion. After taking a Taylor expansion to the INS measurements, $\tilde{z}(t)$ becomes:

$$\tilde{z}(t) = r_{GPS}(t) - \left( r_{INS}(0) + \int_0^t v_{INS}(0)d\tau + \int_0^t \int_0^t a(\tau)d\tau d\sigma + \int_0^t \int_0^t (a'(\tau)\Delta t)d\tau d\sigma \right)$$

$$= z(t) - \int_0^t \int_0^t (a'(\tau)\Delta t)d\tau d\sigma$$

(2.3)

$$= z(t) - \zeta$$

Equation (2.3) shows that the time synchronisation error of the INS measurements introduces an additional observation error $\zeta$. The magnitude of this error is dependant
on the change of the vehicle acceleration $a'(t)$ (also called jerk) which represents the dynamics, and the delay time $\Delta t$. The impact of this error on Kalman filtering results is through the measurement update. Considering the INS synchronisation errors, the measurement update becomes:

$$
\hat{x}_k = \hat{x}_{k-1} + K_k \left( z_k - \zeta_k - H_k \hat{x}_k \right)
$$

(2.4)

The state estimation errors caused by $\zeta$ at each epoch are:

$$
\epsilon = \hat{x}_k - \hat{x}_k = -K_k \zeta_k
$$

(2.5)

This implies that the observation error $\zeta_k$ is distributed to individual state estimates according to the Kalman gain matrix. During each measurement update, this error distribution is largely dependent on the observation geometry $H_k$. Due to the temporal filtering effect of the Kalman filter, the white and Gaussian elements of the error $\epsilon$ would be damped out as the filtering process becomes stabilised. The non-white and Gaussian part would remain as an estimation bias.

The impact of the synchronisation error can be numerically analysed by adding intentional time delays to INS data, and investigating their influence on the integration results. When different increments of time delay are added, the positioning errors steadily increase as the time delay becomes larger. The same trend is observed when the magnitude of the time delay becomes negative, which is the case when the INS clock bias is negative. More detailed results are presented in the following section.

In general, the time synchronisation accuracy required by the integrated GPS/INS system can be evaluated using Equation (2.4). If

$$
|\zeta| << \left| z_k - H_k \hat{x}_k \right|
$$

(2.6)

then the estimation errors caused by the INS data latency can be omitted. From Equation (2.6) and (2.3), one can derive:

$$
|\Delta t| << \left| \frac{z_k - H_k \hat{x}_k}{\int_0^t \int_0^{a'(\tau)} \Delta t d\tau ds} \right|
$$

(2.7)

Some conclusions can be drawn from Equation (2.7). First, requirements on time synchronisation accuracy is negatively proportional to the magnitude of the Kalman
filter innovations, which is largely dependent on INS accuracy and how well it is calibrated. Since higher grade INS will result in smaller innovation magnitude, higher time synchronisation accuracy is required when the INS is to be calibrated using GPS measurements. In contrast, lower time synchronisation accuracy is sufficient when using low grade INS sensors. Second, requirements on time synchronisation accuracy is positively proportional to the magnitude of the vehicle dynamics. Higher time synchronisation accuracy is required for high dynamic applications.

Precise calculation of the required $\Delta t$ using Equation (2.7) may not be possible when considering the simplification in derivation, and the impacts of the errors from INS accelerometers, gyroscopes, and the gravity model. However, a rough estimation can still be given. Suppose a GPS/INS integrated system can have an acceleration change of 9.8 m/sec$^3$ (jerk), and the GPS updating rate is 1 Hz. Double integration of the jerk value generates a speed of approximately 5 m/s. When the innovation RMS is about 5 cm, in order to make the right side value ten times larger than the left side value in Equation (2.7), the required time synchronisation accuracy should be set to 1 millisecond.

### 2.3.2 Numerical analysis

To investigate the impact of time difference on the positioning errors, two sets of ground based experimental data were processed, using AIMS$^\text{TM}$ software. The AIMS$^\text{TM}$ software package was used for GPS/INS integration processing. It was developed by the Center for Mapping at the Ohio State University for large scale mapping and precise positioning applications (Da, 1997; Grejner-Brzezinska, 2004). The first set of data was obtained from the Center for Mapping, OSU, while the second set was collected at the University of New South Wales. The hardware configuration for the system used by OSU for collecting the data consisted of two GPS receivers, and one Litton LN-100 strapdown INS. The configuration used by UNSW consisted of two Leica receivers and one DQI-NP INS. (DQI-NP was used purely as an INS, even though it has a GPS receiver integrated with it.)
Processing results of the original OSU data are illustrated by Figure 2.2, which shows the 3D trajectory of the system.

Figure 2.2 Output trajectory when the OSU original data was processed

Since an accurate reference trajectory cannot be directly measured, the ambiguity-resolved segments are used for comparison in order to assure the accuracy of reference at the centimetre level. Figure 2.3 shows the positioning error in the local North-East-Down frame (NED) when compared with the GPS-only solution. The standard deviation of the positioning errors is within 2cm. These results conform with the reported test accuracy from OSU (Da, 1997; Grejner-Brzezinska, 2004).

Figure 2.3 Positioning error compared with GPS-only results (only the part when ambiguities are resolved)
Then time latency was added to the time tags of the INS data in order to simulate transmission delay. The resulting INS data was re-processed following the same integration procedure as before. As expected, an increase in positioning errors has been observed in all test results, with different time delays injected into the INS data. To demonstrate, Figure 2.4 shows the situation when a 10ms delay was simulated. It can be seen that positioning error increased, but the change did not reach a significant level. 10ms delay only caused the standard deviation of positioning errors to increase to over 2cm. The means of the positioning errors were also increased to more than 2cm.

Figure 2.4 Positioning error compared with GPS-only results when time delay is 10ms

As different increments of time delay were added to the time tags of the INS data, the positioning errors steadily increased as the time delay became larger. The same trend was obvious even when the magnitude of the time delay became negative, as for the case when INS data is transferred faster than GPS data (Figure 2.5). It can be observed that time tag differences of the order of 10ms between GPS and INS data seem tolerable when only positioning error is considered, and when GPS phase ambiguities can be resolved.
In order to investigate the impact of time delay on the trajectory segments when GPS phase ambiguities cannot be resolved; INS errors estimated by the Kalman filter were further studied. In contrast to the segments when GPS phase ambiguities were resolved and where positioning error can be somehow bounded by the high precision GPS measurements, positioning accuracy for segments when the GPS phase ambiguities are unresolved is significantly degraded. The estimated accelerometer biases, scale factors and gyro biases when time delay was added to the INS data were compared with those
when there was no time delay. Figure 2.6 illustrates the differences in the estimation of INS sensor measuring errors from the no-delay estimation versus having delays. The estimation when there was no delay was treated as reference, so the difference is zero at delay time zero.

It can be seen from Figure 2.6 that an increasing time delay has caused the estimated INS errors to deviate quickly from the original estimation when there was no time delay added. For example, the initial estimation of accelerometer bias in the Kalman filter was 3.0e-4 m/s/s which equals to about 30 μg. The technical specification of the INS states 25 μg for the accelerometer bias. The 10ms delay has caused additional estimation error over that amount. This degradation of estimation accuracy may explain the large error when the whole trajectory with 10ms time delay (as shown in Figure 2.7) is compared with the original trajectory (Figure 2.2).

The 10ms delay in the INS data has caused the integration accuracy to be degraded from the centimetre level to the decimetre level when GPS phase ambiguities are not resolved in a short period of time, i.e. max. 111 seconds (Figure 2.8). When GPS phase ambiguity resolution is lost for a longer time, for example the last segment of the trajectory has lost ambiguity resolution for approximately 20 minutes; the resulting trajectory diverged very quickly. (The divergent part is not drawn out in Figure 2.8 in order to show more details of the smaller errors.) It should be noted that the magnitude of the errors demonstrated here is only in a relative sense since the comparison is not based on the true trajectory.
Figure 2.7 Output trajectory when 10ms delay is added to INS data

Figure 2.8 Error of trajectory with 10ms time delay when compared to the original trajectory

Correlation coefficients between velocity trajectories and positioning error trajectories were calculated at different delay times in order to evaluate the influence of vehicle
speed on GPS/INS positioning accuracy. No obvious correlation was detected at any of the tested delay steps (Figure 2.9). Indeed, it is mainly the change of the vehicle acceleration $a'(t)$ (jerk) that influences the positioning accuracy.

![Figure 2.9 Correlation coefficients between velocity and positioning error versus time delay](image)

2.4 SYNCHRONIZATION METHODS

2.4.1 Existing solutions

GPS time is typically used as a time reference for GPS/INS integrated systems. In addition to outputting positioning data and time messages through a serial data link, most GPS receivers provide a 1 pulse-per-second (PPS) electrical signal indicating the time of the turnover of each second. The alignment of the 1PPS signal edge to standard GPS time is normally better than 1µs (Mumford, 2003).

2.4.1.1 Analogue interface

A successful implementation of this scenario is described in detail by (Farrell, 2001b) and (van Grass and Farrell, 2001), and is illustrated in Figure 2.10. The GPS 1PPS signal is used to trigger a 10 kHz 8-channel 16-bit analogue-to-digital (A/D) converter. The A/D converter simultaneously samples the analogue outputs from accelerometers,
gyros and temperature sensors of an ISA. The samples are averaged over 0.01s and sent for GPS/INS data processing.

![Image of time synchronisation scheme](image)

Figure 2.10 Time synchronisation scheme proposed by (van Grass and Farrell, 2001)

A similar solution has been proposed by (Li, 2004), in which a 0.4 millisecond synchronisation accuracy has been achieved using off-the-shelf components. The details concerning the construction of the synchronised A/D sampling circuit have been given by (Ma et al., 2004). Time synchronisation using A/D sampling circuits can be very precise (better than 1μs is possible). The synchronisation between GPS and INS sampling instances eliminates the necessity of doing interpolation during data processing.

However, not all commercial INS sensors provide analogue outputs. Adapting existing INS interfaces to the synchronised A/D sampling circuit is not a trivial task. Even if some manufacturers may provide INS sensors with analogue outputs, such as in the case of the Xbow IMU400CC, the output measurements normally go through an internal pre-processing procedure which includes analogue-to-digital conversion, raw sampling data manipulation, and digital-to-analogue conversions. The delays caused by those processes are hard to estimate.

Without using commercial INSs, individual inertial sensors are available on the market for building up proprietary INSs. However, their usage is largely limited to the lower end applications. This is because there are so many factors limiting them from reaching
a high performance, which might include errors in mounting axes alignment, mounting base deformation, electrical circuit noise, and the lack of high precision calibrations using professional equipment. As will be discussed in section 2.3.1 “Error Propagation”, using a low grade INS might preclude the need to have the same time synchronization accuracy as for high grade INSs.

2.4.1.2. Digital interface

Figure 2.11 shows a conceptual solution proposed by (Knight, 1992) where an INS sensor has a digital data interface. The timing module is designed to accept the GPS 1PPS signal, and the 10.23MHz signal which is generated from the GPS P-code chipping rate. The timing module generates a synchronised 100Hz PPS signal and passes it to the INS sensor. The sampling of inertial sensor measurements is synchronised to this 100Hz PPS signal. The 1PPS signal is also introduced into the integration processor to define the beginning of each second. The INS data can thus be time-tagged according to GPS time received through an RS-422 link. The limitation of this solution is that the INS sensor has to be able to accept GPS PPS signals, which is very unlikely.

![Figure 2.11 A synchronisation solution proposed by (Knight, 1992)](image)

In (Grejner-Brzezinska, 2004; Toth, 2005) a successful implementation using the digital interface to synchronise the Litton LN100, Honeywell HG1700 and Xbow IMU400C (see Figure 2.12) is described. In this scheme, the timing is accomplished by using the high precision PC clock as a common time base. A hardware timer which is running with 1 microsecond resolution provides the link between the GPS time and the PC time. At every transition of the 1PPS signal the hardware saves a record. Through an interrupt, the software samples both the timer and the PC time and stores all values for
later processing. The transformation between the GPS and PC time base is done in two steps: first, GPS vs. Timer; and then, PC vs. timer.

Figure 2.12 Synchronisation scheme introduced by (Grejner-Brzezinska, 2004; Toth, 2005)

One limitation in using the digital interface is the necessity of data interpolation in order to make INS data coincide with GPS data in the Kalman filter measurement update. This is due to the asynchronous measurement sampling of the GPS and the INS sensors. When the INS sampling rate is 100Hz and the GPS sampling rate is 1Hz, a maximum misalignment of 5 milliseconds can only be bridged using an interpolation technique.

Besides a digital interface, some INS sensors also output an electrical PPS signal to indicate the validation of the measurement data (similar to the 1PPS generated by a GPS receiver). With the INS PPS signal, the INS internal pre-processing latencies can be determined and compensated for.

When the INS sensor does not have a PPS signal output, the precise determination of the INS internal processing latency and communication latency is a challenge. Some software methods have been developed to estimate time synchronisation errors in data fusion algorithms (Lee et al., 2002). Nevertheless, a hardware circuit is still necessary to provide initial data alignment; and the time synchronisation accuracy is comparatively low in this case.
Due to the variety of possible time synchronisation scenarios, there are some other non-typical solutions (Ford, 2004; Petrie, 2003; Varley and Maney, 2000) applied under certain situations, which will not be detailed here.

2.4.2 Proposed method

Based on an analysis of the synchronisation scenarios and requirements, three steps are necessary in order to solve the time synchronisation problem:

- Construct a link between the GPS and INS time axes
- Correct the INS time-tag errors
- Interpolate INS data to ensure the INS and GPS measurements coincide at each epoch

Figure 2.13 illustrates the proposed time synchronisation system comprising a GPS receiver, an INS and one integration platform. The integration platform can either be a personal computer, or an embedded processor board. Both the GPS receiver and the INS have two connections to the integration platform, i.e. one serial data link and one electrical PPS signal connection.

![Figure 2.13 Proposed time synchronisation design](image-url)
On the integration platform a series of 100 kHz ticking pulses are generated by a stable source and are counted by a 24-bit counter. The leading edge of the GPS 1PPS signal and INS PPS timing mark triggers the corresponding register to latch the counter. The registers store the value until the next trigger signal comes. The time difference between the GPS 1PPS signal and the INS time mark can be calculated by comparing the values of the two registers.

During the triggering intervals, the stored values in the register are picked up by the serial port interrupt services when their corresponding serial messages are received. Hence the received messages can be tagged with the latched counter values. By referring to those counter values, a link between GPS time and INS time messages can be established.

When the INS PPS timing mark is not available, the manual enable in Figure 2.13 has to be used to set the register into the pass-through mode. In this way, the INS register reading corresponds to the arrival time of the INS message.

When the INS can send a PPS timing mark to the integration platform (as illustrated in Figure 2.13), the true time of the INS PPS signals can be calculated using:

\[
INS_{\text{PPS true time}}_i = GPS_{\text{time sec}}_k + \frac{p_i - n_k}{n_k - n_{k-1}}
\]  
(2.8)

Subscript \(k\) and \(l\) denote the latest GPS and INS sampling epochs, respectively. Variables \(n\) and \(p\) denote the readings of the GPS register and INS register respectively. Thus the INS internal clock bias can be calculated using:

\[
INS_{\text{time bias}} = INS_{\text{PPS time}} - INS_{\text{PPS true time}}
\]  
(2.9)

This calculated INS time bias is used to calibrate the INS data time-tags.

When an INS PPS timing mark is not available, the time that the INS register’s value represents is the INS message arrival time. Hence the INS register readings have included additional delays, including the INS data processing delay and serial communication delay, which have to be extracted. Precise determination of the magnitude of these additional delays is quite challenging. Often the INS manufacturer
would provide it as a constant parameter. Or the parameter can be evaluated experimentally, which is the method adopted in this study and will be demonstrated in the following sections.

With correct time-tags, the raw INS data can be easily interpolated to the points that exactly coincide with the GPS measurements. The interpolation can be linear, quadratic, or cubic, depending on the accuracy required. The interpolation algorithm can be implemented in an Extended Kalman Filter (EKF) time update process.

When the proposed method involves interfacing to the INS analogue outputs, an A/D conversion circuit needs to be added to the integration platform. The trigger signal of the A/D circuit can be used to trigger the INS latching register, as indicated in Figure 2.14.

![Figure 2.14 Time synchronisation with analogue INS inputs](image-url)
2.5 IMPLEMENTATION AND VALIDATION

2.5.1 Implementation

To verify the feasibility of the proposed time synchronization method, a data logging system has been implemented using off-the-shelf components, including a notebook PC, Windows operating system, data acquisition card (DAQ), and the LabView software package from National Instruments. The DAQ card is a low-cost E series multifunction PCMCIA card. It has two 24-bit counters and one 100 kHz internal pulse source. A LabView application was developed to handle the data received from the two EIA-232 ports, i.e. one for receiving GPS data and one for receiving INS data.

![Figure 2.15 Field test configuration](image)

The field tests carried out employ two Leica SR530 GPS receivers, C-MIGITS II (DQI-NP) system including a Jupiter GPS receiver, one Xbow IMU400CC unit. One Leica GPS receiver was set up as a reference station. The antenna of the GPS rover and the INS units were fixed on a mounting board on top of the roof of the test vehicle. The lever arms were precisely surveyed. The other auxiliary devices and logging PC were fixed inside the cabin.

In the test, GPS data from the Jupiter GPS receiver and raw IMU data from the Crossbow IMU400C were time synchronised and logged using the time synchronisation
system. The logged data was then compared with the data set obtained from the C-MIGITS II (DQI-NP).

Since most field tests carried out in this research would employ a similar configuration, for convenience a detailed introduction about the main system elements are given here, which will not be repeated in following chapters.

2.5.1.1 Leica SR530 GPS receiver

Leica GPS SR530 is a 24-channel, dual frequency GPS receiver with on-board RTK capability. Although designed primarily for high accuracy GPS surveying, it can be used in many flexible ways like as a real-time rover for navigation. The collected GPS data can be either stored on the on-board PCMCIA card (PC card), or streamed out in real-time via its serial communication port. With radio modem, the receiver can receive real-time reference data and carry out RTK tasks. Leica Geo-Office is the software tool used for GPS data post processing.

Figure 2.16 Leica SR530 GPS receiver (photos taken from Leica SR530 user manual)
2.5.1.2 C-MIGITS II (DQI-NP)

![Image of C-MIGITS II (DQI-NP) inertial navigation system](image)

Figure 2.17 C-MIGITS II (DQI-NP) inertial navigation system (Boeing, 1997)

C-MIGITS II (DQI-NP) is a tactical grade INS, and belongs to the Miniature Integrated GPS/INS Tactical System (MIGTS™) product family which was originally developed by Rockwell, and then taken over by Boeing (Boeing, 1997; Martin and Detterich, 1997), and now owned by Systron Donner Inertial. DQI-NP is an extension of the C-MIGITS II product, which consists of a solid state tactical grade Digital Quartz IMU (DQI) based on micro machined Quartz technology, and a navigation processor adapter (NPA). The Inertial Sensor Assembly (ISA) of DQI consists of three quartz angular rate sensors (QRS), three vibrating quartz accelerometers (VQA), the drive electronics, preamplifier circuitry for the sensor outputs, and digital conversion electronics. Based on that, the IMU contains additional electronics that process the raw sensor signals for health monitoring and compensation, and provides the changes in attitude and velocity per each sampling period at 100 Hz (also called delta-theta and delta-v) for navigation purpose. The NPA provides input/output (I/O) level conversion and electromagnetic interference (EMI) filtering for interfacing with the DQI. There is also GPS/INS integration software that resides within the DQI.

Connecting the DQI-NP to an external GPS receiver is necessary in order for it to operate properly. The standard data protocol between DQI-NP and external GPS receiver employs the Precision Lightweight GPS Receiver (PLGR) message set. The GPS receiver used in this research is a Rockwell Microtracker, a five-channel, Course/Acquisition (C/A) code, L1 frequency GPS engine. As for outputs, the DQI-NP can provide all essential data for guidance, navigation, and control purpose including...
three-dimensional position and velocity, precise time, attitude, heading, angular rate, and acceleration. The results have a positioning accuracy of 4.5 m (CEP) when the DGPS option is enabled, and 76 m (CEP) when in standard position Service Mode (SA effective). Without SA, the second figure would be better.

DQI-NP provides one full-duplex, asynchronous RS232 serial data port for communication with the host. Optionally, a synchronous RS422 data port can also be used. Besides generating standard three-dimensional navigation solution mentioned above, the pure inertial measurements are available by enabling raw measurement output, which is used in this research for testing optimal integration algorithms.

Table 2.1 C-MIGITS II (DQI-NP) technical specification (Boeing, 1997; Martin and Dutterich, 1997)

<table>
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<th>Gyro channel</th>
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<th>1σ</th>
<th></th>
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<td>deg/h</td>
<td>1σ</td>
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<td>Scale factor stability</td>
<td>ppm</td>
<td>1σ</td>
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<tr>
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<td>Micro-g/root-Hz</td>
<td>max. nom.</td>
<td>180 60</td>
</tr>
</tbody>
</table>
2.5.1.2 Crossbow IMU

The Xbow IMU400CC is a commercial grade IMU, belonging to the DMU™ series motion and attitude sensing units which are manufactured by Crossbow Technology, Inc. (Crossbow, 2002). It is a solid-state six-degree-of-freedom IMU manufactured with MEMS technology. The three angular rate sensors consist of vibrating ceramic plates that utilize the Coriolis force to output angular rate independently of acceleration. The three MEMS accelerometers are surface micro-machined silicon devices that use differential capacitance to sense acceleration. It employs on board digital processing to provide application-specific outputs and to compensate for deterministic error sources within the unit.

Table 2.2 Xbow IMU400CC technical specification (Crossbow, 2002)

<table>
<thead>
<tr>
<th>Gyro channel</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bias</td>
<td>deg/sec</td>
<td>1</td>
</tr>
<tr>
<td>Scale factor stability</td>
<td>%</td>
<td>1</td>
</tr>
<tr>
<td>Angle random walk</td>
<td>deg/root-h</td>
<td>2.25</td>
</tr>
<tr>
<td>Accelerometer channel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bias</td>
<td>milli-g</td>
<td>8.5</td>
</tr>
<tr>
<td>Scale factor stability</td>
<td>%</td>
<td>1</td>
</tr>
<tr>
<td>Velocity random walk</td>
<td>m/s/root-h</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Measurement data can be output via analogue signals and an RS-232 serial link. When using serial communication, it provides two working modes:
• In the voltage mode, the serial data are the raw measurements of the A/D conversion of the sensor analogue signals.

In the scaled sensor mode, the analogue sensor signals are first sampled, converted to digital data, temperature compensated, and scaled to engineering units, then sent out.

2.5.2 Results analysis

Three tests were carried out to verify the time synchronization accuracy of the proposed circuit. First, to verify the accuracy and stability of the timing circuit, the implemented data logging system was used to measure the time intervals of a periodic electrical pulse signal. Second, to verify the timing resolution of the implemented system, the time difference between two electrical pulse signals was measured. Third, the system was used in collecting field data and the results were compared with an independent commercial system.

2.5.2.1 Measurement of periodic timing signals

In this test the time synchronisation system was used to measure GPS 1PPS intervals. Since the GPS 1PPS timing mark is strictly aligned to the GPS second to within ±1μs (Rockwell, 1997), the 1PPS interval is effectively 1s. With 100-kHz ticking frequency, the counting value during each 1PPS interval should be 100k. Repeated tests showed that the actual counting values were either 100k or 100k –1, which is equivalent to 0.01 millisecond timing accuracy.

2.5.2.2 Measurement of two consecutive timing signals

In this test, the source signals are the GPS 1PPS timing pulse and a signal generated by delaying the GPS 1PPS for pre-defined time intervals. These time intervals are controlled in steps of about 20μs. The shortest time interval is 25±10μs. The two source signals were precisely monitored using a digital oscilloscope with a timing accuracy better than 1μs. Figure 2.19 is a snapshot of the oscilloscope display. The blue line (the upper line) is the original GPS 1PPS signal, and the black line (the lower line) is the generated signal. The time interval between the going-up edges of the two signals is the
delay time monitored using the oscilloscope. The same delay time was measured simultaneously using the implemented time synchronisation system.

Comparison of the two results indicates that for every 0.1ms change of the delay time in the range from 0.1ms to 0.9ms, the corresponding counting values changed $10 \pm 2$ (with 100-kHz ticking frequency). So the counting accuracy can be converted into timing accuracy of 0.02ms. To be conservative, the timing accuracy of the implemented circuit is claimed to be better than 0.1ms.

![Oscilloscope image showing the time interval between the going-up edges of two signals](image)

Figure 2.19 Oscilloscope image showing the time interval between the going-up edges of two signals

### 2.5.2.3 Correlation check

With the aid of the GPS receiver, the internal time of the C-MIGITS II is precisely synchronized to GPS time. Through a serial port, the C-MIGITS II can output precisely time-tagged IMU raw measurements at a 100Hz data rate. The IMU400CC data were time synchronised and logged using the implemented time synchronisation system. The IMU400CC accelerometer measurements and the C-MIGITS II reference in the x-direction have been plotted in Figure 2.20. Plots of other measurements have a similar appearance.
Figure 2.20 Comparison between recorded IMU400C raw data and C-MIGITS II data:

Delta V in x-direction

Figure 2.21 Check of Xbow and C-MIGITS II data consistency using the correlation method

It can be observed in Figure 2.20 that the two recorded results coincide with each other very well. To examine the consistency in detail, a cross covariance check of the two data streams was carried out using MATLAB tools. If the two data streams are fully synchronized, the cross covariance peak should appear where the time-lag equals zero. Figure 2.21 shows the result comparing the IMU400CC raw accelerometer x-direction data and the corresponding C-MIGITS II reference data before applying time synchronization corrections. The peak value of the cross covariance appears when the lag equals one, which means the C-MIGITS II data is leading the IMU400C data by
approximately one sampling period. The equivalent time difference is about 10ms when
the data sampling rate is 100Hz.

The comparison of time synchronisation accuracy using the cross covariance method
has been limited by the data sampling frequency, which is 100Hz. Alternatively, the
consistency of the two data streams has been investigated using the cross-coefficients
method. The cross-coefficient of the two independent data series is a normalised
coefficient value which represents the degree of similarity between the two data series.
The cross-coefficient equals one when the two data series exactly repeat each other.

The advantage of using the cross-correlation coefficient method is that several data
series can be generated from the objective data set by shifting time tags by 1
millisecond each time, and then treated as independent data series to be compared with
the reference data. The most similar data sets would have the highest cross-correlation
coefficient value. By examining the time lag where the highest cross-correlation
coefficient appears, the timing difference between the two data sets can be found and be
used for time synchronization compensation. Figure 2.22 shows the results of this
comparison. A 7.5 milliseconds average latency of IMU400CC data is observed from
repeated tests. According to the manufacturer’s technical specification (Crossbow,
2002), the IMU400CC has a data latency of about 6.4 milliseconds. Considering the
platform processing time, the 7.5 milliseconds total time delay is reasonable.

Overall, the time synchronisation accuracy of this field test has been limited by the fact
that the Xbow IMU400CC does not provide precise information about the internal
processing delay, neither through an instantaneous analogue output nor via PPS timing
marks. The synchronisation accuracy in this case is estimated to be at the 2 milliseconds
level based on the distribution of the cross-correlation coefficient peaks. However, the
synchronization accuracy of the proposed hardware circuit has been proved in test 1 and
2, as described above, to be at the level of 0.1ms. Since the typical vehicle jerk for a
passenger car is below 0.01 m/s/s/s and may be up to 4 m/s/s/s level during emergency
break manoeuvres (Wieser, 2007), 2 milliseconds synchronization accuracy is translated
to maximum 4 mm observation error according to Equation (2.7), which is sufficient for
general ground vehicle applications.
Chapter 2  

2.6 SUMMARY

This Chapter has reviewed the time synchronization issue in GPS and INS integration. Factors influencing the accuracy have been identified and analysed. Depending on the requirements of the integration performance and the type of INS sensor used, different time synchronization strategies should be applied. Practical limitations of sensor types and interfaces would influence the choice of time synchronization solutions, and the synchronization accuracy that could in fact be achieved.

To find the optimal time synchronization solution for a particular GPS/INS integration application, first of all the required time synchronization accuracy has to be evaluated properly. This Chapter has proposed an algorithm to analyse the impact of data latency on Kalman filter measurement updates, and hence to determine the synchronization accuracy that should be satisfied.

An innovative time synchronization solution that is simple and flexible for implementation has been proposed and tested using a low-grade MEMS IMU sensor. A time synchronization accuracy of about 2ms has been reached, which is limited by the
nature of the sensor and the interface type. Nevertheless, the feasibility of the proposed solution has been verified.

The analysis results have demonstrated that within the integration Kalman filter, the measuring errors caused by the time synchronization biases are not directly transferred into positioning errors. In this case, to provide an evaluation on how accurate an integrated GPS/INS system should implement the time synchronization mechanism, it is meaningful to make it clear how time synchronization errors propagate. The findings in this thesis point out that depending on the dynamics of a platform, the requirements on synchronization accuracies may vary. Synchronization errors exceeding certain limits would surely degrade the estimation performance.

Although this study is concerned with the integration of a GPS and an INS, the principles are equally applicable to cases when additional imaging and navigation sensors are involved.
CHAPTER 3
AN APPROACH FOR IMU DE-NOISING

3.1 INTRODUCTION

The mechanism of INS error sources is quite complicated and constitutes an independent research field (Titterton and Weston, 1997). As a well known index, the INS navigation errors are exponentially proportional to INS sensor noise level. For GPS/INS integration purpose, the INS sensor noises can be described as having two parts: a low frequency component and a high frequency component (Nassar and EL-Sheimy, 2005). Other classifications are also possible (Chiang et al., 2004). Suppressing the INS noise level is essential for reaching higher performance when using lower cost INS sensors.

Figure 3.1 (a) Power spectrum of INS sensor errors (b) Error deduction by integration and de-noising techniques
When integrated with GPS, the low frequency component of the INS sensor noises can often be modelled with sufficient accuracy as a stochastic process, and be estimated and compensated using complex data fusion techniques like Kalman filter; while the high frequency component is often treated as a white noise process. Since high frequency noise can not be properly modelled, it has to be eliminated or suppressed using de-noising or pre-filtering techniques before they can contaminate the filtering process (El-Rabbany and El-Diasty, 2004; El-Sheimy et al., 2004; Grejner-Brzezinska et al., 2005; Nassar and El-Sheimy, 2006). Conventional de-noising methods include moving average and low-pass filtering techniques. In recent years, wavelet decomposition is more often presented as an effective method to cope with the inertial sensor noise. Several other novel methods, like neural network de-noising (El-Rabbany and El-Diasty, 2004), have also been widely investigated.

One known drawback of using conventional filtering techniques or wavelets methods is that there is no rigorous criterion to evaluate the cut-off frequency or the wavelet de-noising levels. Users need to be very cautious to prevent over-smoothing effects (Grejner-Brzezinska et al., 2005). This over-smoothing may invalidate some of the predetermined models of deterministic errors and stochastic correlated colour noises. To prevent the possible removal of the true dynamics from the signal, the bandwidth of the true motion dynamics must be carefully analysed together with the spectrum characteristics of the wavelet de-noising algorithm.

This Chapter investigates a novel method of using the information of ground vehicle dynamics in de-noising raw INS sensor noises. The concepts of using vehicle dynamic constrains in GPS/INS integration process is not new (Dissanayake et al., 2001; Godha and Cannon, 2005; Wang and Wilson, 2002). Vehicle dynamics can constitute additional observation information for improving the GPS/INS integration performance. Since the vehicle dynamic model has a low pass filter characteristic, passing the raw INS sensor measurements through it would effectively reduce the high frequency noises. This filtering processing has been implemented with a Kalman filter.
3.2 PROPOSED ALGORITHM

3.2.1 Modelling vehicle dynamics

To simplify the study, the ground vehicle discussed here is assumed to be moving without tire side slips, and satisfies the conditions set for an Ackerman model (Gillespie, 1992; Wong, 2001).

\[ m a_x = f_d - f_a - f_r - f_g \]  

where  

- \( m \) is the vehicle mass  
- \( a_x \) is the vehicle longitudinal acceleration  
- \( f_d \) is the vehicle tractive force

Figure 3.2 Simplified vehicle dynamic model

The coordinate system used in Figure 3.2 is the vehicle body frame, which is originated from the vehicle mass centre with x-axis along the vehicle longitudinal axis pointing forwards, y-axis along the lateral direction pointing rightwards, and z-axis perpendicular to the vehicle floor pointing downwards. This coordinate system will be used in the rest discussion of this section.
Chapter 3  An approach for IMU de-noising

\( f_a \) is the vehicle aerodynamic resistance
\( f_r \) is the vehicle rolling resistance
\( f_g \) is the vehicle grade resistance

Since vehicle mass \( m \) in Equation 3.1 is just a scalar coefficient, it is assumed to be unity and omitted in the following discussion.

The translational dynamics in the lateral direction can be written as Equation 3.2 (Gillespie, 1992; Wong, 2001):

\[
a_y = \frac{v_x^2}{R}
\]  

(3.2)

where \( a_y \) is the vehicle lateral acceleration
\( v_x \) is the vehicle forward speed along the longitudinal direction
\( R \) is the vehicle turning radius

Considering the simplification assumptions, the translational dynamics in the vertical direction can be considered as zero, see Equation 3.3.

\[
a_z = 0
\]  

(3.3)

Similarly, the rotational dynamics in two horizontal directions are zero, see Equations 3.4 and 3.5:

\[
\omega_x = 0
\]  

(3.4)

\[
\omega_y = 0
\]  

(3.5)

where \( \omega_x \) is the roll rate
\( \omega_y \) is the pitch rate

The yaw rate of the vehicle can be written as Equation 3.6 (Gillespie, 1992; Wong, 2001):

\[
\omega_z = \frac{v_x}{R}
\]  

(3.6)

where \( \omega_z \) is the yaw rate
\( v_x \) is the vehicle forward speed along the longitudinal direction
Chapter 3  An approach for IMU de-noising

\[ R \] is the vehicle turning radius

The complete ground vehicle dynamic model can be written as Equation 3.7:

\[
\begin{bmatrix}
    a_x \\
    a_y \\
    a_z \\
    \omega_x \\
    \omega_y \\
    \omega_z
\end{bmatrix}
= 
\begin{bmatrix}
    f_d - f_a - f_r - f_g \\
    v_x^2 / R \\
    0 \\
    0 \\
    0 \\
    v_x / R
\end{bmatrix}
\]

(3.7)

And

\[
\dot{v}_x = a_x
\]

(3.8)

Considering Equation 3.8, examining the right hand side of Equation 3.7 shows that such ground vehicle dynamic model has two degrees of freedom, i.e. the vehicle turning radius \( R \), and the vehicle net tractive force \( f_d - f_a - f_r - f_g \) which equals the vehicle longitudinal acceleration \( a_x \). This result coincides with people’s daily driving experience: manoeuvring turning wheel to control the vehicle turning radius to follow road geometry; depressing the vehicle accelerator to overcome the resistance forces to keep the vehicle travelling in a certain speed range.

Generally, both the road turning radius and the longitudinal acceleration are actually random processes for a navigation system. Without detailed physical models, the driving behaviour of a ground vehicle is often modelled as a Gauss-Markov process (Brown and Hwang, 1997), as shown in Figure 3.3 and Equation 3.9. This is a reasonable simplification to reflect the vehicle random behaviours of dynamics by supposing a vehicle is driven on the road near a constant speed where \( a_x \) and \( (1/R) \) are tuned randomly to keep speed and direction as Gauss-Markov processes.

![Gauss-Markov process](image)

Figure 3.3 Gauss-Markov process
The $u(t)$ in Figure 3.3 is unity white noise. The $\beta$, $\sigma$ are the process time constant and the covariance respectively. Expressing a Gauss-Markov process in state space form, we have Equation 3.9:

$$\dot{x} = [-\beta][x] + \left[\sqrt{2\sigma^2 \beta}\right] u(t)$$

(3.9)

For the purpose of implementing the de-noising mechanism, the vehicle dynamic model is relaxed to have full six degrees of freedom, or six independent state variables. Each dynamic state is modelled as a Gauss-Markov process with low frequency characteristic. This is a more generic situation than using strict Ackerman vehicle model. There are two reasons of making this change: first, to overcome some limitations imposed by using the Ackerman vehicle model in real applications; second, since this model is just used for de-noising high frequency noises, it is not necessary to take the risk of violating useful dynamic information. The remaining noises and biases can still be treated in the GPS/INS integration stage.

Rewrite the vehicle dynamic model as the system dynamic matrix of a Kalman filter in a state space form, and treat the disturbance dynamics as process noises, we get Equation 3.10:

$$\begin{bmatrix}
\dot{a}_x \\
\dot{a}_y \\
\dot{a}_z \\
\dot{\omega}_x \\
\dot{\omega}_y \\
\dot{\omega}_z \\
\end{bmatrix} =
\begin{bmatrix}
-\beta_{ax} & 0 & 0 & 0 & 0 & 0 \\
0 & -\beta_{ay} & 0 & 0 & 0 & 0 \\
0 & 0 & -\beta_{az} & 0 & 0 & 0 \\
0 & 0 & 0 & -\beta_{ax} & 0 & 0 \\
0 & 0 & 0 & 0 & -\beta_{ay} & 0 \\
0 & 0 & 0 & 0 & 0 & -\beta_{az} \\
\end{bmatrix}
\begin{bmatrix}
a_x \\
a_y \\
a_z \\
\omega_x \\
\omega_y \\
\omega_z \\
\end{bmatrix} +
\begin{bmatrix}
u_{ax} \\
u_{ay} \\
u_{az} \\
u_{ax} \\
u_{ay} \\
u_{az} \\
\end{bmatrix}$$

(3.10)

where $\beta_{ax}$, $\beta_{ay}$, $\beta_{az}$, $\beta_{ax}$, $\beta_{ay}$, $\beta_{az}$ are time constants, which are used to reflect the low frequency characteristics of the “true” vehicle dynamics.

$w_{ax}$, $w_{ay}$, $w_{az}$, $w_{ax}$, $w_{ay}$, $w_{az}$ are driving white noises as defined in Gauss-Markov model, and are treated as process noises in Kalman filtering.
3.2.2 Observations

When all variables are expressed in vectors under the inertial frame, a triad of accelerometers provides a measure of specific force acting at a point P as Equation 3.11:
\[
\mathbf{f}' = \mathbf{a}' - \mathbf{g}'
\]  
(3.11)
in which \(\mathbf{f}'\), \(\mathbf{a}'\), \(\mathbf{g}'\) are specific force, acceleration, and gravitation vectors under inertial frame (Titterton and Weston, 1997). By pre-multiplying a direction cosine matrix from the inertial frame to the body frame to each vectors in Equation 3.1, we get Equation 3.12:
\[
\mathbf{f}^b = \mathbf{a}^b - \mathbf{g}^b
\]  
(3.12)
where \(\mathbf{f}^b\), \(\mathbf{a}^b\), \(\mathbf{g}^b\) are specific force, acceleration, and gravitation vectors under the body frame of the inertial system. Similarly, a triad of gyros provides a measure of rotation rates \(\mathbf{\Omega}^b\) at point P.

When considering the error models of inertial sensors and Equation 3.11, the accelerometer measurement vector expressed as in Equation 3.13.
\[
\tilde{\mathbf{f}}^b = f^b_{bias} + (1 + S_f) f^b + n_f
\]
\[
= f^b + \left[ f^b_{bias} + S_f f^b + n_f \right]
\]
\[
= a^b - g^b + \left[ f^b_{bias} + S_f f^b + n_f \right]
\]  
(3.13)
and the gyro measurements in Equation 3.14,
\[
\tilde{\mathbf{\Omega}}^b = \Omega^b_{bias} + (1 + S_{\Omega}) \mathbf{\Omega}^b + n_{\Omega}
\]
\[
= \Omega^b + \left[ \Omega^b_{bias} + S_{\Omega} \mathbf{\Omega}^b + n_{\Omega} \right]
\]  
(3.14)
where \(\tilde{\mathbf{f}}^b\) is the raw measurements of specific force vector
\(f^b_{bias}\) is the bias vector of raw accelerometer measurements
\(S_f\) is the scale factor error vector of raw accelerometer measurements
\(n_f\) is the random noise vector of raw accelerometer measurements
\(\tilde{\mathbf{\Omega}}^b\) is the raw measurements of angular rate
\(\Omega^b_{bias}\) is the bias vector of raw gyro measurements
\(S_{\Omega}\) is the scale factor error vector of raw gyro measurements
\( \mathbf{n}_n \) is the random noise vector of raw gyro measurements

In reality, external factors may cause additive disturbances to those vehicle dynamics like road bumps, stones, vehicle engine vibrations and many other high frequency disturbances from unnamed sources. Compared with those dynamics with genuine navigational interests, these disturbance dynamics are generally having high frequency characteristics. These disturbance dynamics are defined as Equation 3.15:

\[
\mathbf{w}_a = \begin{bmatrix} w_{ax} \\ w_{ay} \\ w_{az} \end{bmatrix}
\quad (3.15)
\]

and Equation 3.16

\[
\mathbf{w}_{\omega} = \begin{bmatrix} w_{\omega x} \\ w_{\omega y} \\ w_{\omega z} \end{bmatrix}
\quad (3.16)
\]

For navigation purpose, inertial sensors are rigidly fixed to the vehicle body to measure the vehicle dynamics. It is reasonable to assume that the inertial system body frame is fully aligned with the vehicle body frame, i.e. the inertial x-axis to the vehicle x-axis, the inertial y-axis to the vehicle y-axis, the inertial z-axis to the vehicle z-axis. So inertial sensor measurements have following relationship with the vehicle dynamics, see Equation 3.17:

\[
\mathbf{a}^b = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \mathbf{w}_a
\quad (3.17)
\]

And when the Earth rate in angular rate measurements is omitted, see Equation 3.18

\[
\mathbf{\Omega}^b = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \mathbf{w}_{\omega}
\quad (3.18)
\]

Hence from Equations 3.13, 3.14, 3.17, 3.18, a complete observation matrix can be written as:
The Equation 3.19 also constitutes the observation equation of the Kalman filter.

The \( \mathbf{g}^b \) in Equation 3.12 can be calculated by Equation 3.20 (Titterton and Weston, 1997):

\[
\mathbf{g}^b = \mathcal{C}_n^b \mathbf{g}^n = \begin{bmatrix}
\sin \theta \\
-\sin \phi \cos \theta \\
-\cos \phi \cos \theta
\end{bmatrix} \mathbf{g}
\]

(3.20)

Where \( \theta, \phi \) denote pitch and roll angles and \( \mathbf{g} \) is the gravitational constant.

Assuming the vehicle is moving on a near horizontal surface, the \( \mathbf{g}^b \) can be simplified as in Equation 3.21,

\[
\mathbf{g}^b = \begin{bmatrix}
0 \\
0 \\
-1
\end{bmatrix} \mathbf{g}
\]

(3.21)

Based on Kalman filtering mechanism, high frequency observation noises \( \mathbf{w}_a, \mathbf{w}_\omega \) and \( \mathbf{v}_f, \mathbf{v}_\Omega \) would be suppressed after filtering process. Since the biased components in the raw accelerometer and gyro measurements can not be eliminated at this stage, the estimation of the vehicle dynamics are actually biased, as can be seen from Equation 3.19. These low frequency errors from inertial sensors left in the state estimations will be handled during the next stage when inertial measurements are blended with GPS observations.

3.3 TEST AND RESULTS

3.3.1 Evaluation of dynamic parameters
Just for test purposes, existing position, velocity and attitude trajectories obtained from GPS/INS integration are used to evaluate the magnitude of the actual ground vehicle dynamics. Then the vehicle dynamic information, i.e. standard deviations of vehicle acceleration and angular rates, is used to set up stochastic modelling parameters of the Kalman filter. In real applications, this information would be obtained through accumulative collection of road experience.

Since the uncertainty of the dynamic vehicle trajectory from the GPS ambiguity resolved solution is within several centimetres, acceleration profiles are calculated by differencing the velocities of two consecutive epochs using Equation (3.22).

\[
a_{\text{NED},k} = \sqrt{a_{N,k}^2 + a_{E,k}^2 + a_{D,k}^2} = \sqrt{(v_{N,k} - v_{N,k-1})^2 + (v_{E,k} - v_{E,k-1})^2 + (v_{D,k} - v_{D,k-1})^2} \tag{3.22}
\]

Here, \(a_{N}, a_{E}, a_{D}\) and \(v_{N}, v_{E}, v_{D}\) represent the vehicle acceleration and velocity under local NED frame. While \(a_{\text{NED}}\) is the total acceleration, so its magnitude should be equivalent to the specific force sensed by the IMU sensor under its body frame. Although the result is noisy, it is sufficient for evaluating the magnitude of the dynamics.

The angular rates are calculated by differencing the heading and attitude measurements. When the IMU is rigidly affixed to the vehicle body, the IMU x-axis is pointing forward, the IMU y-axis is pointing rightward, the IMU z-axis is pointing downward, the angular rates on the IMU z-axis are mainly associated with the heading dynamics, the angular rates on the x-axis are mainly associated with roll dynamics while the angular rates on the y-axis are mainly associated with pitch dynamics. In this way, an estimation of the vehicle dynamics sensed by the IMU sensors in the body frame during the road test can be roughly estimated, see Table 3.1. The accuracy of the figures in Table 3.1 can be improved after more road data has been collected. In real applications, the figures may be adjusted according to different road and vehicle conditions.
3.3.2 Evaluation of measurement noise

Assume all noise sources, like IMU sensor noise, engine vibration, road bumps, are all independent from each other. Then the noise covariance should have additive characteristics. The total dynamic energy sensed by IMU sensors can be expressed as the variance of the measurement sequences. By subtracting the true vehicle dynamics, the remaining part should represent the noise level. The mean of the measurements have been extracted beforehand. The results have been illustrated in Table 3.2.

<table>
<thead>
<tr>
<th></th>
<th>$f_x$</th>
<th>$f_y$</th>
<th>$f_z$</th>
<th>$\omega_x$</th>
<th>$\omega_y$</th>
<th>$\omega_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>STD</td>
<td>$&lt;0.46$ m/s/s</td>
<td>$&lt;0.46$ m/s/s</td>
<td>$&lt;0.46$ m/s/s</td>
<td>$&lt;0.087$ rad/s</td>
<td>$&lt;0.0024$ rad/s</td>
<td>$&lt;0.0024$ rad/s</td>
</tr>
</tbody>
</table>

Since having the precise value are not possible at current stage, a safe margin is given when setting up the Kalman filtering parameters, which guarantees only the unwanted high frequency noises being filtered out.

3.3.3 Time constant determination

Using a Gauss-Markov model allows a great simplification of sophisticate vehicle models and driving dynamics, meanwhile addressing the uncertainties of the random processes like road conditions and one’s driving behaviour. However, proper determination of the time constant for the Gauss-Markov model may not be a straightforward procedure. In this research, this parameter has been investigated between the range of 0.1 second and 10 seconds, which is based on intuitive experience and analysis of the road geometry, see Figure 3.4.
In the following presented results, a time constant of 5 seconds has been used in modelling acceleration channels, and 15 seconds in modelling angular rates. However, optimised modelling parameters should be based on long term observation of road driving data and systematic analysis of that data, which is beyond the capacity of current research.

![Autocorrelation analysis of the GPS derived accelerations (ambiguity fixed)](image)

Figure 3.4 Autocorrelation analysis of the GPS derived accelerations (ambiguity fixed)

### 3.3.4 Power spectrum analysis

![Power spectrum analysis](image)
Figure 3.5 Sensor measurements before and after filtering
Figure 3.6 Power Spectrum of the sensor measurements before and after filtering

Figure 3.5 and 3.6 shows the INS sensor measurements and their power spectrum before and after dynamic model based filtering has been applied. A good suppression of high
frequency noises have been demonstrated in both the time domain and the frequency domain.

### 3.3.5 Integration results

Figure 3.7 shows the integration results before and after the dynamic model based filtering has been applied.

![Graphs showing integration results before and after filtering](image)

**Figure 3.7** Integration results before and after filtering

### 3.4 SUMMARY

To prevent inertial sensor noises contaminating the integration process, the raw measurements are normally filtered by a low-pass filter. Many new methods have been developed in recent years trying to improve the filtering performance so that low end
MEMS sensors can be used for positioning and navigation applications. However, most methods rely on offline batch processing, to experience some limitations.

To overcome the limitations of existing methods, this Chapter has tested a novel method of using the low pass characteristic of vehicle dynamic models to aid the de-noising process. The parameters of this new method have clear physical meanings, and can be evaluated from existing road geometry and speed profile data. In this case, the cumulative knowledge of the vehicle dynamics can contribute to the improvements of de-noising performance. Since this algorithm is implemented using a standard Kalman filter, it could be combined with the main integration filter to implement online de-noising. The parameters used for de-noising could also be online estimated and adapted to different application environments.

The initial test with real field data have demonstrated clear effects of reducing the inertial sensor noise levels. Although there is no indication of significant performance improvement on final integration results, equivalent accuracies have been achieved. Since at current stage the parameters for both dynamic model and stochastic model are not optimised, further performance improvement is expected as more research work being carried out.
4.1 INTRODUCTION

In this chapter, two adaptive filtering methods have been investigated. One is covariance based on-line stochastic modelling method. Its performances have been evaluated using ground vehicle test data. To overcome the disadvantages of existing methods, this thesis has developed a new method which is called covariance based process noise scaling method.

The central challenge of successfully implementing a GPS/INS integration system is to effectively blend GPS and INS data together in order to generate the optimum solution. Despite various integration architectures, the Kalman filter (KF) is still the most common technique for carrying out this task. The operation of the KF relies on the proper definition of the dynamic model and the stochastic model (Brown and Hwang, 1997). The dynamic model describes the propagation of system states over time. The stochastic model describes the stochastic properties of the system process noise and observation errors.

The uncertainty in the covariance parameters of the process noise (Q) and the observation errors (R) has a significant impact on Kalman filtering performance (Grewal and Andrews, 1993; Grewal et al., 2001; Salychev, 2004). Q and R directly influence the weight that the filter applies between the existing process information and the latest measurements. Errors in any of them may result in the filter being suboptimal or even cause it to diverge.

The conventional way of determining the Q and R requires good a priori knowledge of the process noise and measurement errors, which typically comes from intensive empirical analysis. When accurate application model is available, which is also called “true model”, finding the optimal Q and R settings can be done thorough Monte Carlo analysis. Monte Carlo analysis refers to the process of evaluating the actual system
Chapter 4 Adaptive Kalman filtering algorithms

performance with a simulation of the application process. The simulation of the application process is evaluated over expected trajectories with additive stochastic inputs. When it is applied to an integrated GPS/INS system, the navigation performance can be analysed statistically and compared with the theoretical performance calculated from Kalman filter state covariance matrix (Farrell and Barth, 1998). Hence optimal Q and R values can be acquired. However, a true model and its simulation system are not always available.

In practice, the Q and R values are generally fixed and applied during the whole application segment. The performance of the integrated systems suffers in two respects due to this inflexibility. First, process noise and measurement errors are dependent on the application environment and process dynamics. For generic applications, the settings of the stochastic parameters have to be conservative in order to stabilise the filter for the worst case scenario, which leads to performance degradation. Second, the so-called KF “tuning” process is complicated, often left to a few “experts”, and thus hampers its successful application across a wider range of fields.

Over the past few decades, adaptive KF algorithms have been intensively investigated to reduce the influence of the Q and R definition errors. Popular adaptive methods used in GPS/INS integration can be classified into several groups, such as covariance scaling, multi-model adaptive estimation, and adaptive stochastic modelling (Hide et al., 2004a). The covariance scaling method improves the filter stability and convergent performance by introducing a multiplication factor to the state covariance matrix. The calculation of the scaling factor can either be fully empirical or based on certain criteria derived from filter innovations (Hu et al., 2003; Yang, 2005; Yang and Gao, 2006; Yang and Xu, 2003). The multi-model adaptive estimation method requires a bank of simultaneously operating Kalman filters in which slightly different models and/or parameters are employed. The output of multi-model adaptive estimation is the weighted sum of each individual filter’s output. The weighting factors can be calculated using the residual probability function (Brown and Hwang, 1997; Hide et al., 2004b). Adaptive stochastic modeling method includes innovation-based adaptive modeling (Mohamed and Schwarz, 1999) and residual-based adaptive modeling (Wang, 2000; Wang et al., 1999). By online monitoring of the KF innovation or residual covariances, the adaptive
stochastic modeling algorithm estimates directly the covariance matrices of process noise and measurement errors, and “tunes” them in real-time.

In this Chapter the online stochastic modeling method is first investigated for GPS/INS integration purposes, specifically with regards to convergence, optimal window size, and the interaction between Q and R estimation. The algorithm was implemented on a well-established GPS/INS integration software platform. The results from several field tests have been evaluated against the results obtained using conventional Kalman filter architectures.

Besides the successful use of stochastic modelling method, one observed limitation is that the estimation algorithm is very sensitive to coloured noises and changes in the number of observed satellites. Theoretically, this sensitivity is mainly due to two reasons. First, the covariance estimation of the innovation and residual sequences is normally very noisy due to the short data sets, the coloured noises, and the non-stationary noise property during a short time span. On the other hand, smoothing covariance estimation by increasing the estimation window size would degrade the dynamic response of the adaptive mechanism, and may cause violation of the stationarity assumption. Second, with a limited number of “rough” covariance observations it is difficult to derive precise process noise and observation error estimates. Considering the large matrix dimension of process noise when the INS Psi model is used, a full estimation of the Q and R matrices is very challenging.

Hence, an adaptive algorithm with fewer estimable parameters is desirable for practical use. It is well known that the innovation and residual sequences of the KF are a reliable indicator of KF filtering performance. For an optimal filter, the innovation and residual sequences should be white Gaussian noise sequences with zero mean (Brown and Hwang, 1997; Mehra, 1970). By comparing the covariance estimates of innovation and residual series with their theoretical values computed by the filter, the status of the filter operation can be monitored.

In the second part of this Chapter, a new covariance matching based process noise scaling algorithm is proposed. Without using artificial or empirical parameters, the
The proposed adaptive mechanism has the capability of autonomously tuning the Q matrix to the optimal magnitude. This proposed algorithm has been validated using road test data. A comparatively better filtering performance has been achieved.

4.2 ONLINE STOCHASTIC MODELLING

4.2.1 Algorithms

4.2.1.1 Estimation of $R$

As introduced in chapter 1, the innovation sequence of a Kalman filter is defined as:

$$d_k = z_k - H_k \hat{x}_k$$  \hspace{1cm} (4.1)

The adaptive stochastic modelling algorithm can be derived with the covariance matching principles. Substituting the KF measurement model into Equation (4.1) gives

$$d_k = (H_k x_k + v_k) - H_k \hat{x}_k$$  \hspace{1cm} (4.2)

As pointed out earlier, the innovation sequences $d_k$ is a white Gaussian noise sequence with a zero mean when the filter is in optimal mode. Taking variances (same as autocorrelation here) on both sides of Equation (4.2),

$$E\{d_k d_k^T\} = E\left[\begin{bmatrix} H_k (x_k - \hat{x}_k) + v_k \\ H_k (x_k - \hat{x}_k) + v_k \end{bmatrix}^T \right]$$

$$= E\left[\begin{bmatrix} H_k (x_k - \hat{x}_k) \\ H_k (x_k - \hat{x}_k) \end{bmatrix}^T \left[ H_k (x_k - \hat{x}_k) \right] + \left[ H_k (x_k - \hat{x}_k) \right]^T v_k + v_k \left[ H_k (x_k - \hat{x}_k) \right]^T + v_k v_k^T \right]$$  \hspace{1cm} (4.3)

Considering the orthogonality between observation error and state estimation error:

$$E\{d_k d_k^T\} = E\left[ H_k (x_k - \hat{x}_k) \left[ H_k (x_k - \hat{x}_k) \right]^T \right] + E\{v_k v_k^T\}$$

$$= H_k E\left\{ (x_k - \hat{x}_k) (x_k - \hat{x}_k)^T \right\} H_k^T + E\{v_k v_k^T\}$$  \hspace{1cm} (4.4)

where $x_k$ is the true state value, and $\hat{x}_k$ the predicted state. The covariance (autocovariance) of their difference is the KF predicted covariance matrix $\hat{P}_k$. So,

$$E\{d_k d_k^T\} = H_k \hat{P}_k H_k^T + E\{v_k v_k^T\}$$

$$= H_k \hat{P}_k H_k^T + R_k$$  \hspace{1cm} (4.5)
When the innovation covariance $E\{d_id_i^T\}$ is available, assuming $\hat{P}_k$ is known, the covariance of the observation error $R$ can be estimated directly from Equation (4.5).

The detailed derivation of the estimation algorithms of process noise $Q$ can be found in (Mohamed and Schwarz, 1999; Wang et al., 1999). In brief, adaptive estimation of $R$ is actually linked with the process noise $Q$ due to the fact that the derivation is based on the Kalman filtering process. This can be seen from Equation (4.5), that in order to estimate $R$, the calculation of the predicted state covariance $\hat{P}_k$ has used $Q$. Further numerical analyses are presented later, but in general reliable estimation of $Q$ and $R$ at the same time using an adaptive estimation method will not always be feasible (Maybeck, 1982; Wang et al., 1999). The normal practice is to fix one, say $Q$ as defined by the INS error characteristics given by the manufacturers, and estimate the other.

4.2.1.2 Covariance calculation

Calculation of the innovation covariance $E\{d_id_i^T\}$ is normally carried out using a limited number of innovation samples (Maybeck, 1982; Mohamed and Schwarz, 1999; Wang et al., 1999):

$$E\{d_id_i^T\} = \frac{1}{m} \sum_{i=0}^{m-1} d_{k-i}d_{k-i}^T$$  \hspace{1cm} (4.6)

where $m$ is the ‘estimation window size’. For Equation (4.6) to be valid, the innovation sequence has to be ergodic and stationary over the $m$ steps.

The estimation converges to the true value as the window size becomes larger, and if the assumptions of an ergodic and stationary stochastic process remain valid. However, large window sizes and the fulfilment of the stationary condition are contradictory requirements in kinematic GPS/INS applications. Hence choosing an appropriate window size is a trade-off between estimation stability and estimation accuracy. Since the autocorrelation estimation deals with a statistical process, there will always be some statistical uncertainty in the results. The practical choice of the estimation window size may depend on the system dynamics.
4.2.2 Implementation and test

4.2.2.1 Test Configuration

The test configuration is similar as described in Chapter II. The DQI-NP’s specified parameters were used in setting up the Q matrix in the Kalman filtering process. Since GPS carrier phase ambiguities are resolved during the whole time span, the measurement errors are considered to be about 0.01m. This value is used for setting up R in the standard EKF filtering.

Test data were collected and post processed using self programmed integration software which is mainly implemented on the MATLAB platform. Commercial GPS software toolbox like GPSoft have been purchased to support basic functional calls. AIMS™ GPS/INS integration software was often used to generate reference results. In this particular work, the AIMS™ software was modified to test the adaptive stochastic modelling algorithms. This software package was developed by the Center for Mapping at the Ohio State University for large scale mapping and precise positioning applications (Da, 1997; Grejner-Brzezinska, 2004).

4.2.2.2 Influence of the window sizes

To illustrate the influence of different window sizes on R estimation, the square roots of the R estimations (RMS of the measurement errors) using the window sizes of 128, 256, 384, 512 and 640 are illustrated in Figure 4.1. Only the value corresponding to the first diagonal element of R is shown here (corresponding to the first double-differenced measurement), however the remaining estimation curves are similar.
All estimations with different window sizes converged to a value of about 0.05m. The estimation oscillation becomes obvious when shorter window sizes are used, such as 128. Window sizes shorter than 128 epochs have also been tested, but showed much greater oscillation characteristics. This confirms the findings of an earlier analysis that a short window size may make estimation unstable.

Starting with a window size of 384 and larger, the estimation stabilised quickly after switching over from the initial values, and the overall estimates of R become much smoother. In this sense, window sizes of 384 or 512 are good candidates for setting up the adaptive filter.

The step jump of the estimation trajectories are due to the switch over from initial settings when the estimation algorithm is put into operation. During the first period of the window size, the default Q and R values were used.

Figure 4.2 shows the RMS trajectories of the adaptive Kalman filter’s state covariance matrix - which is an indicator calculated by the Kalman filter for measuring the filtering
accuracy and stability as discussed earlier. Here only the first diagonal element has been shown. The trends of the remaining state covariances are similar. The results show that the overall filter operation is stable and converged, no matter the estimation window size used.

Figure 4.2 RMS of the estimated state covariance when R is estimated using different window sizes (corresponding to the first diagonal element of the state covariance matrix)

4.2.2.3 Influence of the initial settings

To investigate the effectiveness of the adaptive estimation algorithm, initial values Q and R were set intentionally with a very large bias to simulate the situation when only very approximate a priori knowledge of the system is available. During the test, the initial RMS values of the measurement errors have been changed from 0.01m to 0.1m. For clarity, only three RMS values of the R diagonal elements are plotted in Figure 4.3 (corresponding to the estimated measurement errors). The window size used for the R estimation is 384.
Figure 4.3 Influence of biased initial values on RMS of R estimation (a) SQRT of R(1,1) (b) SQRT of R(2,2) (c) SQRT of R(3,3)

Figure 4.3 shows that different initial values of Q and R have a large impact on the performance during the transit period. However, the initial estimation deviations are damped quickly using the proposed on-line stochastic modelling procedure. The estimation converges with time.

The above results show that without precise a priori knowledge of R, the adaptive estimation algorithm can estimate its values in a stable fashion. The requirement of a priori knowledge of the stochastic model has been largely removed. Whilst using a large window size may lead to a more stable and smoother R estimation, it may also prolong the transit period when a large bias appears in the initial values.

4.2.2.4 Overall performance

It is well known that the state covariance matrix in a Kalman filter indicates the filtering performance. However, the covariance values may be overestimated or underestimated if Q and R are not properly defined.
An alternative method is to investigate the covariance of state estimation directly. Since their true means are known to be zero (KF states are actually the errors to be estimated), if the whole system was perfectly modelled in the Kalman filter and the estimation was in a stable state, state estimation should be a zero-mean white noise process. Unlike the state covariance matrix generated by the Kalman filter self, this directly calculated state covariance is a reliable filtering quality measure. The square roots of the diagonal elements of the calculated state covariance are the standard deviations (STD) of the estimated states.

The STDs of state estimation from direct calculation have been plotted in Figure 4.4 against the different window sizes that were used. The STDs are generally smaller when the adaptive estimation method has been used. The STDs are reduced further as the estimation window size is increased. But after a window size of 512, the improvement becomes minor.
Figure 4.4 Comparison of filtering performance with different window sizes (a) position (b) velocity (c) attitude
4.2.2.5 Influence of Q on the estimation of R

Since the adaptive estimation of Q and R is not always feasible (as discussed above), the normal procedure of adaptation is to fix one value while estimating another. To investigate the influence of different values of Q on R estimation, a series of R estimates were obtained with Q changing from one, ten and one hundred times its default value. The window size used for the estimation is 512. As an example, Table 4.1 shows the square root values of the estimation of the first diagonal element of R at epoch 900. Initial STD for the GPS measurements was set as 0.01m. Table 4.2 shows the corresponding filtering accuracies of the estimated states.

The results show that when the Q values are increased, the values of R estimation have a tendency to decrease. However, the estimation accuracy may degrade because of the estimation deviation from the true covariance of the measurement errors. When initial values of Q are varied across a reasonable range, its influence on R estimation and Kalman filtering outputs becomes negligible.

So in a practical implementation of on-line adaptive stochastic modelling, the Q can be set according to the manufacturers’ data. On-line estimation of R can produce a reliable stochastic model for the Kalman filter.

Table 4.1 SQRT of the estimated R(1,1)

<table>
<thead>
<tr>
<th></th>
<th>Q</th>
<th>Q*10</th>
<th>Q*100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated</td>
<td>0.032m</td>
<td>0.028m</td>
<td>0.025m</td>
</tr>
</tbody>
</table>

Table 4.2 STD of estimated state x(1)

<table>
<thead>
<tr>
<th></th>
<th>Q</th>
<th>Q*10</th>
<th>Q*100</th>
</tr>
</thead>
<tbody>
<tr>
<td>STD</td>
<td>0.022m</td>
<td>0.027m</td>
<td>0.038m</td>
</tr>
</tbody>
</table>
4.3 COVARIANCE BASED ADAPTIVE PROCESS NOISE SCALING

4.3.1 Algorithms

To improve the robustness of the adaptive filtering algorithm, a new process noise scaling method is proposed here.

For an optimal filter, the predicted innovation covariance should be equal to the one directly calculated from the innovation sequence, as illustrated by Equation (4.6). Any deviation between them can be ascribed to the wrong definition of $\hat{P}_k$ and/or $R_k$ in Equation (4.7). Considering that the Kalman gain $K_k$ is dependent on the relative magnitudes of $\hat{P}_k$ and $R_k$, and $R_k$ has several other ways to be assessed for GPS/INS integration performance, $R_k$ is assumed to be perfectly known for adaptation purposes. So,

$$\frac{1}{m} \sum_{i=0}^{m-1} d_{k,i} d_{k,i}^T = H_k \hat{P}_k H_k^T + R_k \quad (4.7)$$

where $\hat{P}_k$ denotes the estimation of the process noise prediction. Since attempting to directly resolve $\hat{P}_k$ from Equation (4.7) is not practical (although a partial adaptation might be possible), to simplify, a scaling factor is defined as:

$$\alpha = \frac{\text{trace} \left\{ H_k \hat{P}_k H_k^T \right\}}{\text{trace} \left\{ H_k \hat{P}_k H_k^T \right\}} = \frac{\text{trace} \left\{ \frac{1}{m} \sum_{i=0}^{m-1} d_{k,i} d_{k,i}^T - R_k \right\}}{\text{trace} \left\{ H_k \hat{P}_k H_k^T \right\}} \quad (4.8)$$

where the scaling factor $\alpha$ implies a rough ratio between the calculated innovation covariance and the predicted one, since

$$\hat{P}_k = \Phi_{k-1} \hat{P}_{k-1} \Phi_{k-1}^T + Q_{k-1} \quad (4.9)$$

By substituting Equation (4.9) into Equation (4.8), $\alpha$ can be expressed as:

$$\alpha = \frac{\text{trace} \left\{ H_k \left( \Phi_{k-1} \hat{P}_{k-1} \Phi_{k-1}^T + \hat{Q}_{k-1} \right) H_k^T \right\}}{\text{trace} \left\{ H_k \left( \Phi_{k-1} \hat{P}_{k-1} \Phi_{k-1}^T + Q_{k-1} \right) H_k^T \right\}} \quad (4.10)$$

Based on the same principle used for covariance matching (Mehra, 1970; Wang et al., 1999), from Equation (4.8) and Equation (4.10), an adaptation rule is defined as:

$$\hat{Q}_k = Q_{k-1} \sqrt{\alpha} \quad (4.11)$$
The square root in Equation (4.11) is used to contribute a smoothing effect. Directly tuning $\hat{P}_k$ based on Equation (4.8) is not considered viable due to the concerns of filtering smoothness and parameter consistency.

Compared with the existing covariance scaling methods, the distinct features of this proposed algorithm are: 1) the adaptive algorithm is not applied to tuning the state covariance matrix $\hat{P}_k$; 2) the factor $\alpha$ can be a scaling factor either larger than one or smaller than one, which provides a full range of options to tune $\hat{Q}_k$. Only when the predicted innovation covariance and the calculated innovation covariance are consistently equal, does $\alpha$ then stabilise at the value of one.

The innovation covariance still needs to be estimated using Equation (4.6). When compared with the adaptive stochastic modelling method, the process noise scaling method is more robust to covariance estimation bias due to fewer parameters being involved in the tuning, and the tuning process is a smooth feedback. However, since only the overall magnitude of $\hat{Q}_k$ is tuned rather than individual elements, optimal allocation of noise to each individual source cannot be achieved. This is one fundamental difference between the adaptive stochastic modelling methods and the covariance scaling methods.

### 4.3.2 Test results

Figure 4.5 shows the ground track of the test vehicle. Figure 4.6 shows the RMSs of the adaptive Kalman filter’s states derived from the covariance matrix $\hat{P}_k$. Only the RMS values of the first three diagonal elements have been shown. The trends for the remaining states are similar. The overall filter operation is stable and converged. The “bump” at about 100 epochs is caused by the switching over to the adaptive algorithm. The window size used for the innovation covariance calculation is 64.
Figure 4.5 Ground track of the test vehicle

Figure 4.6 RMSs of the adaptively estimated Kalman filter states

Figure 4.7 shows the histogram of the estimated scaling factor with time. As expected, after the transition period the scaling factor quickly settles to a value of about one.
For the optimal Kalman filter, both innovation and residual magnitudes should be minimised. Figure 4.8 (a) shows that the magnitude of the innovations is within 0.1m. After measurement update, the magnitude of the residuals is within 0.02m, as illustrated in Figure 4.8 (b). Since the necessary and sufficient condition for the optimality of a Kalman filter is that the innovation sequence should be white, the autocorrelation of the innovation sequence is plotted in Figure 4.9, which clearly shows white noise features. A further check of the whiteness can be carried out using the method introduced by (Mehra, 1970).
Figure 4.8 (a) Innovation sequence (b) Residual sequence
Figures 4.10 and 4.11 show two groups of accelerometer bias and gyro bias estimates for comparison purposes. The process noise parameters used by the conventional extended Kalman filter are calculated according to the manufacturer’s technical specification. It can be seen that all three configurations have generated similar estimates. The conventional extended Kalman filter provides the smoothest estimation. The estimates using the process scaling method are much noisier, which may be due to its quick response to signal changes. The estimates on the Z axis have the worst consistency. This may be due to its weak observability, since during the ground vehicle tests the Z axis has the least dynamics. Another reason could be that gravity uncertainties were not properly scaled. They may behave differently from the INS noises.
Chapter 4 Adaptive Kalman filtering algorithms

(a)

(b)
Figure 4.10 Estimates of accelerometer bias using different methods: (a) Standard Extended KF (b) Extended KF with online stochastic modelling (c) Extended KF with the proposed process scaling algorithm.
Figure 4.11 Estimates of gyro bias using different methods: (a) Standard Extended KF (b) Extended KF with online stochastic modelling (c) Extended KF with the proposed process scaling algorithm
The observability of a linear system in a Kalman filter context depends on the system
dynamic matrix and observation matrix (Bar-Itzhack, 1988; Goshen-Meskin and Bar-
Itzhack, 1992a; Goshen-Meskin and Bar-Itzhack, 1992b; Hong et al., 2005; Wang,
2000; Wang et al., 1999). Change of the stochastic modelling parameters will not
influence the observability of the system. However, changes of the noise covariance
matrix do influence the filter convergence speed and estimation accuracy. It may
quicken or slow down the estimation process, or even cause the filter to diverge.

As to the proposed process noise scaling method, when the adaptation process becomes
stable (which is indicated by the scale factor being stabilized at one, see Figure 4.7), the
process noise covariance matrix is actually kept unchanged in the following estimation
iterations. This reduces the adaptive extended Kalman filter to the conventional
extended Kalman filter.

4.4 SUMMARY

Over the past few decades adaptive KF algorithms have been intensively investigated
with a view to reducing the influence of the uncertainty of the covariance matrices of
the process noise (Q) and the observation errors (R). The covariance matching method
is one of the most promising techniques.

Many theoretical and numerical analyses have shown that the process noise covariance
matrix Q and the measurement error covariance matrix R have significant influences on
the Kalman filtering performance, especially for applications of integrated GPS/INS
systems in demanding dynamic environments. Since proper definition of the Q and R
values are not trivial tasks, and flexibility is necessary for multi-sensor systems to adapt
to different application environments, an adaptive estimation method is an attractive
solution.

This Chapter has investigated the adaptive online stochastic modelling method in the
context of a tightly-coupled GPS/INS integration scheme. The algorithms have been
tested on a well-established GPS/INS integration platform. The results have shown that
for ground-based vehicle applications, the adaptive estimation of Q and R in a tightly-coupled GPS/INS integration system with extended Kalman filter can deliver stable and reliable filtering results. An estimation window size of about 500 epochs may be optimal to deliver smooth Q and R estimates.

However, the practical implementation of an online stochastic modelling algorithm faces many challenges. One critical factor influencing the stochastic modelling accuracy is ensuring precise and smooth estimation of the innovation and residual covariances from data sets with limited length. Furthermore, the stochastic parameters are closely correlated with each other when using current estimation algorithms, which make correct estimation more difficult. The online adaptive stochastic modelling method is not scalable for the estimation of a large number of parameters.

Due to the above considerations, a new covariance-based adaptive process noise scaling method has been proposed and tested. Initial tests have shown that this method has better tolerance to modelling inaccuracy, and hence delivers comparatively better results. However, the optimal allocation of noise to each individual source is not possible using scaling factor methods.
CHAPTER 5
A NEW METHOD TO INTEGRATE GPS AND MEMS INS

5.1 INTRODUCTION

Due to the high cost of the sensors, GPS/INS integrated systems is conventionally targeted at higher end applications, such as mobile mapping and imaging, remote sensing, air-borne gravimetry and so on. However, with the advance of geo-spatial technology, the situation has changed dramatically in recent years.

Firstly, GPS is becoming smaller and cheaper. A consumer-level GPS chip set can be easily fitted into a PDA or mobile phone, with cost no more than 10 Australian dollars. Secondly, the price of INS sensors especially MEMS sensors has also been dropping down quickly. In view of the fast development of MEMS technology, the quick reduction of MEMS sensor cost, and the continuous improvements in MEMS sensor accuracy, it is widely believed that MEMS INS could eventually replace the conventional INS sensors in middle to lower end applications, like car navigation, unmanned-autonomous-vehicle (UAV) navigation, micro-robot control and other location based services. In brief, there is an emerging trend of using GPS/INS integrated systems for real-time low-cost applications.

To fulfil this demand, there are some challenges existing that need further investigation. Due to the low cost, low grade INS sensors especially MEMS sensors are not performing as well as conventional high-grade INS sensors in view of their significant scale factor nonlinearities, large mounting misalignment errors, high electric noise and temperature sensitivity. During operation, regular calibrations using external aiding sources are essential to limit their rapidly growing errors.

GPS is an ideal aiding source to this purpose, as has been demonstrated so far in previous Chapters. However, high GPS positioning accuracy is not easily achievable in low-cost and real-time applications. In a benign environment with full operational GPS capability, a stand-alone GPS receiver can deliver positioning solutions with uncertainty
as low as several meters. With real-time support of correctional data, pseudo-range based differential GPS (DGPS) can provide positioning accuracy of sub-meter level for real-time applications. To further exploit higher accuracy from GPS measurements, GPS carrier phase ambiguities have to be resolved on the fly, which normally requires dual frequency receiver, real-time differential infrastructure support such as continuous operational reference stations, and additional communication facilities between reference and rover stations. Under the name of Real Time Kinematic positioning (RTK), this technology is obvious unsuitable for real-time low-cost applications. Furthermore, resolving GPS carrier phase integer ambiguity is a complex task, limited to distances below 50km between reference and rover, and not always reliable.

Without fixing carrier phase integer ambiguities, the GPS position aiding accuracy would be limited to approximately several meters in standalone mode, and to the sub-meter level in DGPS mode. Although positioning information with such accuracy may be sufficient to bound INS navigation error growth, it is unlikely that the INS sensor errors will be calibrated precisely, especially when MEMS INS are involved in the integration process. This is fundamentally due to the fact that when the Kalman filter (KF) observations become nosier (observation covariance becomes larger), it takes a longer time for the KF state estimation to converge to a certain accuracy. Meanwhile, the low grade MEMS INS, with poor performance, has a much shorter error modeling time constant when compared with higher grade conventional INS. For a navigation-grade INS, its time constant is normally at several hours; for a tactical-grade INS, it is about several minutes; in the case of MEMS INS, it is even shorter. In this case, the filter may fail to track INS error states properly if the estimations could not converge quickly, especially for those states with weak observability such as the attitude states.

To address the above issues, this Chapter investigate an innovative solution in which MEMS INS errors have been calibrated using carrier phase derived velocity information from a stand-alone GPS receiver. The overall structure of this implementation includes two calibration steps. The inner loop estimates and corrects IMU errors using delta positions between GPS epochs. The outer feed forward loop bounds the integrated position and attitude error growth using GPS pseudorange measurements so as to
eliminating the bias of the inner filtering loop. The algorithms described here are suitable for real-time applications.

5.2 INTEGRATION ARCHITECTURE

5.2.1 Two-step aiding

To avoid the need for fixing carrier phase ambiguities, GPS velocity measurements can be adopted as an alternative aiding source (Moafipoor et al., 2004; Wendel et al., 2006; Wendel and Trommer, 2004). Since Doppler derived GPS velocity is normally considered to be too noisy, aiding with GPS carrier phase derived velocity is dominant. In (Moafipoor et al., 2004), GPS rover velocity has been calculated from the double differenced carrier phase rate, and then the velocity is used as the observations in a tightly coupled implementation. In (Wendel et al., 2006; Wendel and Trommer, 2004), time-differenced carrier phase measurements from standalone receivers are used. An observation relation between the direct time-differenced carrier phase and estimated velocity states involves a time integral component which complicates the process.

Inspired by the segmented integration scheme introduced by (Farrell, 2001a; Farrell, 2002; Farrell, 2006; van Grass and Farrell, 2001), a two step calibration structure has been proposed as illustrated by Figure 5.1. Similar to the solution of van Grass and Farrell (van Grass and Farrell, 2001), the integration includes a high dynamic calibration loop and a low dynamic PVA calibration loop. In the inner feed-back and feed-forward loops associated with the first EKF estimator, delta positions between consecutive GPS epochs sensed by the INS are compared with corresponding GPS measurements which can have up to one centimeter accuracy (the accuracy will be discussed later) to generate the IMU error calibrations and corrections of the INS only PVA navigation results. Since only relative position information is used in the inner loop, accumulated filtering biases may exist in the velocity calibrated PVA (see Figure 5.1) due to the integration effect. The outer feedforward loop aims to bound the velocity calibrated PVA error growth using GPS pseudorange measurements so as to eliminate the low frequency drifting bias of the inner loop outputs.
This approach has the following advantages:

- INS sensor errors can be more precisely calibrated so that not only the INS standalone performance is improved, but also attitude estimation accuracy would be improved. Hence as an attitude sensor, MEMS INS becomes both more reliable and more accurate.

- After the high frequency errors are filtered out from the PVA solution, the GPS pseudorange measurements become adequate to filter out the low frequency bias so that the PVA feedback to INS navigation calculation can guarantees the filter’s stability.

This solution is based on the previous study of van Grass and Farrell (van Grass and Farrell, 2001), which includes a high dynamic calibration loop and a low dynamic PVA calibration loop. Since temporal disposition measurements are used in the high dynamic calibration loop, and absolute position measurements are used for a low dynamic calibration loop, the system implementation in Chapter 5 actually comprises two position aiding loops. Due to convenience for implementation, the integration has been separated into two separate position aiding loops. Combining these two loops into one Kalman filter would require additional adjustments of system observation mechanism.
5.2.2 Velocity aiding versus delta position aiding

Using a GPS receiver as a velocity and acceleration sensor has attracted a lot of research interests due to the potential applications in navigation, control, and gravimetry studies (Jekeli and Garcia, 1997; Serrano et al., 2004a; Serrano et al., 2004b), as well as being used as an additional aiding source in GPS/INS integration.

In general there are three methods to derive velocity from GPS measurements:

- Differencing positions
- Using GPS Doppler measurements
- Using time-differenced GPS carrier phase measurements

Amongst the three methods, the time-differenced carrier phase method is the most favourable one in view of the achievable accuracy. Previous studies show that a few millimetres per second accuracy is achievable under static test conditions, and up to several centimetres in kinematic mode which is dependent on receiver quality, raw measurement calibration and the vehicle dynamic level (Ryan et al., 1997). In a reported flight test, differences between the stand-alone velocity solution and the post processed differential velocity solution were found to be at the 2-4mm/s level (1 sigma) for horizontal velocity components and 9.7 mm/s (1 sigma) for vertical velocity (van Grass and Farrell, 2001). First order central difference approximation of the carrier phase is reported to be the most superior observations in calculating the GPS (Ryan et al., 1997).

However, direct comparison of the velocities calculated through the above methods with INS velocities might not be the best choice for integration, because the GPS velocities are actually the average velocity measured across one or several GPS sampling periods. The GPS velocities may differ from the instantaneous true velocity in such a way that when the host vehicle dynamic is greater, the difference is larger. On the other hand, the INS normally has a higher sampling frequency (say 100Hz). So its velocity represents the velocity measurements of a much shorter time interval. When GPS and INS measurements are fused at each GPS epoch, they are not directly comparable if higher precision is required.
In (Moafipoor et al., 2004), with the assumption that the absolute location is known at
the starting epoch, the velocity observations have been converted into the position
coordinates using the following equation:

\[ \bar{x}_{t+\Delta t} = \bar{x}_t + \bar{v}_t \Delta t \]  \hspace{1cm} (5.1)

Where \( \bar{x}_t \), \( \bar{x}_{t+\Delta t} \) denotes the positions at two consecutive epochs, \( \bar{v}_t \) the velocity, \( \Delta t \) the
sampling period. There are two limitations with this solution:

- Calculating delta positions from either average or instantaneous velocity by
  multiplying the time period is not accurate.
- Velocity errors may accumulate in the integrals. A numerical demonstration of
  these errors will be presented in the following data analysis.

Due to the above considerations, GPS velocities or GPS velocity derived positions are
not treated as aiding sources in the proposed filtering process. Rather the delta positions
measured by GPS and INS are compared epoch by epoch (GPS epoch). For our
purpose, a more rigorous algorithm for calculating delta positions from time-differenced
carrier phase measurements has been derived and presented in the following Section.
Since the delta position is the distance a GPS receiver moves within one sampling
period (1 second here), it has the same magnitude as the average velocity. The average
velocity here means averaged value across a second period. In contrast, some velocity
algorithms introduced in the literacy are actually calculating average velocities across a
period several seconds long. Despite this difference, the term “GPS velocity” may still
be used hereafter in the discussion instead of “delta position” for convenience.

### 5.2.3 Delta distance algorithm

The velocity algorithm described in the Section will calculate the rigorous GPS receiver
displacement between two GPS epochs that are not necessarily consecutive to each
other. This feature brings the convenience that epochs with bad or missing
measurements can simply be skipped. This is the situation that is often encountered
under canopy or in an urban canyon. When the delta positions calculated in this manner
are used for INS aiding, the INS calculation period should be adjusted accordingly.
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The derivation of the algorithm is based on implementing a Taylor expansion on two GPS observations at different epochs at one common initial position. The initial position can be simply GPS pseudorange based solutions. The numerical analysis shows that several tens of meters positioning errors would cause velocity determination errors of less than one centimetre.

By differencing the expanded GPS observations, sorting out the velocity algorithm is straightforward. Pre-compensation of Iono, Tropo and satellite clock errors are essential in achieving high precision velocity measurements.

![Diagram](image)

**Figure 5.2** Calculation of delta distance based on GPS carrier phase measurements

GPS carrier phase measurement

\[
\lambda \phi_i^j (t) + c \delta_i^j (t) = \rho_i^j (t) + \lambda N_i^j + c \delta_i^j (t) + I_i^j (t) + T_i^j (t) + M_i^j (t) + \varepsilon
\]

(5.2)

where \( \phi_i^j (t) \) is the measured carrier phase expressed in cycles, \( \lambda \) is the wave length, \( \rho_i^j (t) \) is the geometry distance between the satellite and the observation point, \( N_i^j \) is the time independent phase integer ambiguity, \( \delta_i^j (t), \delta_i^j (t) \) are satellite and receiver clock...
biases, $I'_i(t)$ is the ionosphere error, $T'_i(t)$ is the troposphere error, $M'_i(t)$ is the multipath error, $\varepsilon$ is the combined residual error, and $c$ is the light speed constant. The superscript denotes satellite ID, and the subscript denotes receiver ID. Time $t$ represents the measurement epoch.

After pre-compensation of iono and tropo errors and satellite clock correction, do temporal difference between epoch $t_1$ and $t_2$,
\[
\lambda \phi'_i(t_2) - \lambda \phi'_i(t_1) = \rho'_i(t_2) - \rho'_i(t_1) + \varepsilon \tag{5.3}
\]
where other common mode errors are assumed to be largely cancelled, the remaining part has been included in $\varepsilon$.

The geometry distance between satellite at time $t_s$ and the observation point at $t_r$ can be calculated in the ECEF frame using:
\[
\rho'_i(t_s, t_r) = \sqrt{(x'_i(t_s) - x_i(t_r))^2 + (y'_i(t_s) - y_i(t_r))^2 + (z'_i(t_s) - z_i(t_r))^2} \tag{5.4}
\]
Where $(x'_i(t_s), y'_i(t_s), z'_i(t_s))$ denotes the satellite position in ECEF frame, and $(x_i(t_r), y_i(t_r), z_i(t_r))$ the observation point. Linearize Equation (5.4) at the position of the observation point at time $t_{r_0}$
\[
\rho'_i(t_s, t_r) = \rho'_i(t_s, t_{r_0}) + \frac{\partial \rho'_i}{\partial x_i} \bigg|_{(x_{r_0}, y_{r_0}, z_{r_0})} \Delta x_i(t_r) + \frac{\partial \rho'_i}{\partial y_i} \bigg|_{(x_{r_0}, y_{r_0}, z_{r_0})} \Delta y_i(t_r) + \frac{\partial \rho'_i}{\partial z_i} \bigg|_{(x_{r_0}, y_{r_0}, z_{r_0})} \Delta z_i(t_r) \tag{5.5}
\]
In (A.4), $(\Delta x_i(t_r), \Delta y_i(t_r), \Delta z_i(t_r))$ is the delta position relative to the position $(x_i(t_{r_0}), y_i(t_{r_0}), z_i(t_{r_0}))$. The three partial derivatives form the unit vector pointing from the SV towards the initial position $(x_i(t_{r_0}), y_i(t_{r_0}), z_i(t_{r_0}))$. 

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Similarly, the geometry distances between satellite and the observation point at time \( t_{r1} \) and \( t_{r2} \) can be linearized, assuming \( t_{r0}, t_{r1} \) and \( t_{r2} \) are not far from each other.

\[
\rho^i_j(t_{ts}, t_{r1}) = \rho^i_j(t_{ts}, t_{r0}) + \left. \frac{\partial \rho^i_j}{\partial x^i} \right|_{(t_{ts}, t_{r0})} \Delta x^i(t_{r1}) + \left. \frac{\partial \rho^i_j}{\partial y^i} \right|_{(t_{ts}, t_{r0})} \Delta y^i(t_{r1}) + \left. \frac{\partial \rho^i_j}{\partial z^i} \right|_{(t_{ts}, t_{r0})} \Delta z^i(t_{r1}) \tag{5.7}
\]

\[
\rho^i_j(t_{ts}, t_{r2}) = \rho^i_j(t_{ts}, t_{r0}) + \left. \frac{\partial \rho^i_j}{\partial x^i} \right|_{(t_{ts}, t_{r0})} \Delta x^i(t_{r2}) + \left. \frac{\partial \rho^i_j}{\partial y^i} \right|_{(t_{ts}, t_{r0})} \Delta y^i(t_{r2}) + \left. \frac{\partial \rho^i_j}{\partial z^i} \right|_{(t_{ts}, t_{r0})} \Delta z^i(t_{r2}) \tag{5.8}
\]

Assume the observation point moved a distance \((\Delta x^i(t_{r2}, t_{r1}), \Delta y^i(t_{r2}, t_{r1}), \Delta z^i(t_{r2}, t_{r1}))\) from time \( t_{r1} \) to \( t_{r2} \)

\[
\begin{align*}
\Delta x^i(t_{r2}) &= \Delta x^i(t_{r2}, t_{r1}) + \Delta x^i(t_{r1}) \\
\Delta y^i(t_{r2}) &= \Delta y^i(t_{r2}, t_{r1}) + \Delta y^i(t_{r1}) \\
\Delta z^i(t_{r2}) &= \Delta z^i(t_{r2}, t_{r1}) + \Delta z^i(t_{r1}) \tag{5.9}
\end{align*}
\]

Differencing Equation (5.7) and (5.8) to get the delta geometry distance \( \Delta \rho^i_j(t_{r2}, t_{r1}) \) between observation time \( t_{r1} \) and \( t_{r2} \).
\[ \Delta \rho'_i (t_{r2}, t_{r1}) = \rho'_i (t_{s2}, t_{r0}) - \rho'_i (t_{s1}, t_{r0}) \]
\[ + \left[ \frac{\partial \rho'_i}{\partial x_i} \right]_{(t_{s2}, t_{r0})} \Delta x_i (t_{r1}) \]
\[ + \left[ \frac{\partial \rho'_i}{\partial y_i} \right]_{(t_{s2}, t_{r0})} \Delta y_i (t_{r1}) \]
\[ + \left[ \frac{\partial \rho'_i}{\partial z_i} \right]_{(t_{s2}, t_{r0})} \Delta z_i (t_{r1}) \]
\[ \text{So,} \]
\[ \Delta \rho'_i (t_{r2}, t_{r1}) = \rho'_i (t_{s2}, t_{r0}) - \rho'_i (t_{s1}, t_{r0}) \]
\[ + \left[ \frac{\partial \rho'_i}{\partial x_i} \right]_{(t_{s2}, t_{r0})} \Delta x_i (t_{r1}) \]
\[ + \left[ \frac{\partial \rho'_i}{\partial y_i} \right]_{(t_{s2}, t_{r0})} \Delta y_i (t_{r1}) \]
\[ + \left[ \frac{\partial \rho'_i}{\partial z_i} \right]_{(t_{s2}, t_{r0})} \Delta z_i (t_{r1}) \]
\[ (5.10) \]
\[ + \left[ \frac{\partial \rho'_i}{\partial x_i} \right]_{(t_{s2}, t_{r0})} \Delta x_i (t_{r2}, t_{r1}) + \left[ \frac{\partial \rho'_i}{\partial y_i} \right]_{(t_{s2}, t_{r0})} \Delta y_i (t_{r2}, t_{r1}) + \left[ \frac{\partial \rho'_i}{\partial z_i} \right]_{(t_{s2}, t_{r0})} \Delta z_i (t_{r2}, t_{r1}) \]
\[ (5.11) \]

If the accurate observer position at time \( t_{r1} \) is known, the delta position between observation time \( t_{r1} \) and \( t_{r2} \) has the following relation to the delta geometry distance:

\[ \Delta \rho'_i (t_{r2}, t_{r1}) = \rho'_i (t_{s2}, t_{r0}) - \rho'_i (t_{s1}, t_{r0}) + \left[ \frac{\partial \rho'_i}{\partial x_i} \right]_{(t_{s2}, t_{r0})} \Delta x_i (t_{r2}, t_{r1}) \]
\[ + \left[ \frac{\partial \rho'_i}{\partial y_i} \right]_{(t_{s2}, t_{r0})} \Delta y_i (t_{r2}, t_{r1}) + \left[ \frac{\partial \rho'_i}{\partial z_i} \right]_{(t_{s2}, t_{r0})} \Delta z_i (t_{r2}, t_{r1}) \]
\[ (5.12) \]

If the observer position at time \( t_{r1} \) is not perfectly known, for example when the GPS pseudo range code derived position is used, the induced error in range domain is
\[
\delta \rho = \left( \frac{\partial \rho / \partial x_i}{\delta x_i} \right) \delta x + \left( \frac{\partial \rho / \partial y_i}{\delta y_i} \right) \delta y + \left( \frac{\partial \rho / \partial z_i}{\delta z_i} \right) \delta z
\]

A numerical evaluation of the error magnitude based on Equation (5.13) has been presented in the data analysis. Since it is a range domain error along the line of sight, DOP factors should be considered. For the error budget of other error sources, (van Grass and Soloviev, 2004) is a good reference for more detailed analysis. Cycle slip detection is not explicitly considered here. As pointed out earlier, missing one calibration step of the inner loop would not have much impact on system performance.

5.3 VALIDATION AND TEST

Integration results of standalone GPS with the Xbow IMU400 using a conventional pseudo range code solution has been illustrated by Figure 5.3. Two large deviations from the reference trajectory are obvious which are caused by satellite geometry changes. Peak heading errors are about 20 degrees in low dynamic test loops and more than 100 degrees during high dynamic maneuvers.
Figure 5.3 Standalone GPS integrated with Xbow IMU400 using only pseudo range solutions (a) ground track (b) Heading

Figure 5.4 shows the GPS velocity calculated from time-differenced carrier phase measurements using the proposed velocity algorithms. Figure 5.5 shows the results when the GPS velocities (GPS delta position of each second) are compared with the delta positions calculated from the GPS carrier phase ambiguity resolved solutions. Table 5.1 provides the comparison statistics. A STD accuracy of approximately less than two centimeters has been reached.

Table 5.1 STD of displacement calculation errors

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>STD</td>
<td>0.020 m</td>
<td>0.012 m</td>
<td>0.012 m</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.001</td>
<td>-0.002 m</td>
<td>-0.0005 m</td>
</tr>
</tbody>
</table>
Figure 5.4 GPS receiver velocity calculated using the proposed algorithm

Figure 5.5 Comparison of the GPS velocity with the delta positions calculated from the GPS carrier phase ambiguity resolved solutions

When GPS velocity errors are integrated, trajectories of relative positioning errors are formed, as illustrated by Figure 5.6. These errors may bias the filtering outputs of the inner velocity aiding loop. The outer filtering loop with the pseudorange measurements aiding would filter these errors out.
Using Equation (5.13), the velocity errors caused by inaccurate initial positions can be evaluated since the line-of-sight (LOS) unit vectors are available during velocity calculation. When a 10 meters X-direction initial position error is considered, the errors caused in LOS range domain have been illustrated in Figure 5.7. The magnitude of the influence depends on the satellite geometry, and was stable during the test period. Since the maximum value is less than 1 mm, the error term Equation (5.13) can be safely omitted in the velocity estimation.
Pre-filtering of MEMS INS raw measurements can effectively reduce its short term error influences in the data fusion process (Chiang et al., 2004). Based on an analysis of the power spectrum in the frequency domain, a moderate low-pass 6-order least squares FIR filter has been utilized to filter the MEMS INS raw data with pass band limits at 25Hz. Figure 5.8 shows the power spectrum of the Xbow IMU400 accelerometers measurements in the X-direction when it is in static mode, kinematic mode and after-filtering kinematic mode. It can be seen the high frequency component has been moderately damped. The de-noised Xbow IMU400 measurements are used for integration.

![Figure 5.8 Power spectrums of the Xbow IMU400 accelerometer measurements](image)

In Figure 5.9, the integrated results using the proposed two-step calibration scheme has been presented. The improvement on the positioning accuracy is obvious. The positioning errors caused by a change in the GPS satellite geometry have been successfully filtered out. The horizontal positioning errors have been limited to within 2 meters even during high dynamic figure eight shaped maneuvers. Some improvements on the attitude and heading accuracies can also be observed. During the low dynamic test segment, the heading error has been limited to within 10 degrees; pitch and roll to within one degree. During the high dynamic test segment, pitch and roll errors are still limited to around 2 degrees; however, heading errors increases to more than 50 degrees.
Chapter 5  A new method to integrate GPS and MEMS INS

(a)

(b)
Figure 5.9 Integrated results using the proposed two-step calibration scheme (a) ground track (b) position error (c) attitude error

The observability issue of error states in an integrated GPS/INS system in an unconstrained three dimensional space has been intensively investigated in the literature (Bar-Itzhack, 1988; Goshen-Meskin and Bar-Itzhack, 1992a; Goshen-Meskin and Bar-Itzhack, 1992b; Hong et al., 2005; Rhee et al., 2004). The total observability of error states relies on the vehicle manoeuvres. With only position aiding by a single antenna GPS receiver, horizontal accelerations are needed in order to properly estimate the yaw error state. For a ground vehicle system that moves with slow changes in attitude and acceleration, the attitude of the vehicle is unobservable with a single-antenna GPS measurement; moreover, the component of yaw bias is not observable if the gyro has a large error (Hong et al., 2005). Thus, the yaw error may increase significantly fast with time.

Velocity vector estimates based on GPS Doppler measurements (or differential carrier phase measurements) can be used to determine the direction of the platform ECEF velocity vector. However, as pointed out by Farrell and Barth (1998), this direction should not related to platform axes direction (i.e. azimuth of heading), as the vehicle heading and velocity vector directions do not necessarily coincide.
Since a low grade MEMS IMU, i.e. Xbow IMU 400CC, was used in the test discussed in Section 5.3, it is expected that heading estimation would have a poorer result. There are several possible methods under investigation in improving the heading estimation in a integrated system, such as introducing into the system additional sensors like magnetometer.

5.4. SUMMARY

This Chapter presents a new approach when only standalone GPS is available for integration with MEMS INS. GPS velocities derived from time-differenced carrier phase measurements have been introduced as an additional aiding source into the data fusion process. A two-step filtering structure has been proposed in the integration scheme in order to improve the high dynamic response, while maintaining the unbiased filtering performance. The inner loop estimates and corrects IMU errors using delta position information between GPS epochs. The outer feed forward loop bounds the overall position and attitude error growth using GPS pseudorange measurements and eliminates the low frequency filtering bias of the inner loop.

GPS velocity determination algorithms have been derived in a slightly different form versus existing methods in order to generate more accurate delta position measurements between GPS epochs. The influence of inaccurate initial positions on velocity determination has been expressed by a formula and evaluated numerically using the field test data. The results have demonstrated that its influence is at the noise level which can be safely omitted in velocity calculation. The overall accuracy of the velocity calculated from time-differenced carrier phase has reached less than two centimetres (STD) when it is compared with GPS ambiguity resolved solutions.

After implementation of the velocity algorithms and the new integration structure, the data processing results show that both the position and attitude estimation accuracies have been improved significantly. For future work, the MEMS INS error model will be further investigated and the adaptive Kalman filtering technique (Ding et al., 2007) will be incorporated into the current integration structure in order to handle the MEMS INS modelling uncertainties.
CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

With the success of GPS/INS integrated systems in high end applications like direct geo-referencing for aerial survey, mapping and remote sensing, there is strong interest in using such systems in lower end applications like car navigations, unmanned autonomous vehicle controls, and personal navigations. In this trend, there have appeared new technical challenges in terms of affordability, usability, reliability, and real-time performance. To respond to these challenges, research of this thesis has been conducted on four specific topics, namely time synchronization issues in constructing a integration hardware platform, de-noising of the INS raw measurements when low cost sensors are used, adaptive Kalman filtering for the integrated system to operate with uncertainties in priori settings, and the integration strategies for a standalone GPS integrated with a low cost MEMS INS.

6.1.1 A method for time synchronization of sensors

Design of a synchronised data logging system is a fundamental task in building a hardware platform for multi-sensor data fusion. Errors in time synchronization would propagate into the data fusion stage and degrade the integration performance. This issue has been investigated both theoretically and empirically in the literature; however a clear definition of the required synchronization accuracies regarding the application dynamics is not available. Also some methods introduced in the literature are not easy to implement, and are not suitable for the purposes of building a generic integration platform.

Chapter 2 has reviewed the limitations and advantages of various time synchronisation scenarios and existing solutions. An important fact has been revealed that in the data fusion stage, the measurement errors caused by the time synchronization biases are not directly transferred into positioning errors. Using the error propagation rules derived in
the research, an example is given about the estimation of the required synchronization accuracies regarding different application dynamics. Then an innovative time synchronisation solution that is simple and flexible has been proposed. To test the proposed solution, it is implemented with off-the-shelf components and the LabView software platform, and used to synchronize the measurements from Xbow IMU400CC. The results are compared with those collected from C-MIGITS II (DQI-NP). The tests show that a time synchronisation accuracy of about 2ms has been achieved, which is limited by the nature of the sensor and the interface type. The resolution and accuracy analysis shows that the proposed solution has the capability to reach time synchronisation accuracy at 0.1ms level if INS could provide a hard-wired timing signal.

In practice, different time synchronisation strategies may be applied depending on the requirements of the integration performance and the type of INS sensor used. Practical limitations of sensor types and interfaces would influence the choice of time synchronisation method, and the synchronisation accuracy that could in fact be achieved. Although this study is regarding the integration of GPS and INS, the principles are equally applicable to cases where additional imaging and navigation sensors are involved.

### 6.1.2 An approach for IMU De-Noising

Low cost INS sensors, especially MEMS INS would exhibit much larger sensor noises than conventional navigation grade or tactical grade inertial navigation systems. In order to use low cost MEMS sensors for navigation and geo-referencing applications, reducing the noise level of the INS raw measurements by de-noising techniques is essential.

In the literature, de-noising methods include using conventional low pass filters, wavelets filters and neural networks where one common limitation of those techniques is that there is no rigid criterion to evaluate the cut-off frequency or the wavelet de-noising levels. Empirical experience and careful analysis are necessary to prevent under smoothing or over-smoothing effect.
The research work in Chapter 3 investigates the plausibility of using vehicle dynamics in pre-filtering (de-noising) process in order to avoid an arbitrary determination of cut-off frequency for de-noising. Based on the analysis of the vehicle dynamics and INS sensor noises, a novel method is proposed. Compared with existing de-noising methods, the parameters of the proposed method have physical meanings and can be directly evaluated from existing trajectory data sets. In this case, the existing knowledge of the vehicle dynamics can contribute to the improvements of the de-noising performance. Implemented as a standard Kalman filtering process, this algorithm is suitable for real time applications.

6.1.3 Adaptive Kalman filtering algorithms

In a Kalman filter, an optimal estimation of the state variables at each epoch is based on the propagation of the dynamic equations from the previous epoch, and new observations. The balance between the contributions of these two information sources is controlled by stochastic models of the process noises and observation noises, and uncertainties of the initial state estimates. Using incorrect stochastic models can cause the data fusion to be sub-optimal, or even diverge.

In conventional GPS/INS integration the stochastic models, namely the estimations of the covariance matrices of the process noises and the observations noises, are normally determined based on intensive analysis of empirical data and kept unchanged during real operations. However, research evidences have shown that the noise levels of both GPS and INS measurements may change in different applications, especially when the integrated systems operates in a high dynamic environment. Also in practice, the GPS/INS integrated system need to have the flexibility to autonomously adapt to any changes of the application environments instead of relying on manual tuning by specialists. This motivates the investigation of using adaptive Kalman filtering for GPS/INS integration.

First in Chapter 4, the adaptive online stochastic modelling method in the context of a tightly coupled GPS/INS integration scheme was investigated. The results have shown
that for ground vehicles, the adaptive estimation of $Q$ and $R$ with extended Kalman filter can deliver stable and reliable filtering results. An estimation window size of about 500 epochs may be optimal to deliver smooth $Q$ and $R$ estimates. The research also shows that the practical implementation of an online stochastic modeling algorithm faces many challenges. One critical factor influencing the stochastic modeling accuracy is ensuring precise and smooth estimation of the innovation and residual covariances from data sets of limited length. Furthermore, the stochastic parameters are closely correlated with each other when using current estimation algorithms, which make correct estimation more difficult. The online adaptive stochastic modeling method is not scalable for the estimation of a large number of parameters.

Due to above considerations, a new covariance-based adaptive process noise scaling method has been proposed and tested. This method has shown to be reliable, robust, and suitable for practical implementation. Initial tests have demonstrated improvements of the filtering performance. However, one common drawback of using scaling factor methods is that an optimal allocation of noises to each individual source is not possible.

### 6.1.4 A new method to integrate GPS and MEMS INS

In view of reducing the overall cost of GPS/INS integrated systems, the integration of a standalone GPS with MEMS INS is of great research interest. One challenge in such a configuration is to use standalone GPS positioning results, which normally have an uncertainty above several meters, to calibrate the MEMS INS errors. Since the MEMS INS errors have a much shorter time constant than higher grade INSs, the estimation of those error states in the data fusion process has to converge quickly in order to enable proper tracking. To improve the filter convergence performance, better accuracy of the aiding measurements is preferred.

Starting from that consideration, research in Chapter 5 investigates the possibility of using precise GPS velocities (actually, it is the precise position change during each GPS sampling period) derived from time-differenced carrier phase measurements as an additional aiding source to improve the INS error calibration.
A two-step filtering structure has been proposed in the integration scheme in order to improve the high dynamic response, while maintaining the unbiased filtering performance. The inner loop estimates and corrects IMU errors using delta position information between GPS epochs. The outer feed forward loop bounds the overall position and attitude error growth using GPS pseudorange measurements and eliminates the low frequency filtering bias of the inner loop.

GPS velocity determination algorithms have been derived in a slightly different form versus existing methods in order to generate more accurate delta position measurements between GPS epochs. The influence of inaccurate initial positions on velocity determination has been expressed by a formula and evaluated numerically using the field test data. The results have demonstrated that its influence is at the noise level which can be safely omitted in velocity calculation. The overall uncertainty of the velocity calculated from time-differenced carrier phases has reached less than two centimetres per second (STD) when it is compared with a GPS solution with ambiguities resolved.

After implementation of the velocity algorithms and the new integration structure, the data processing results show that both the position and attitude estimation accuracies have been improved significantly.

6.2 RECOMMENDATIONS FOR FUTURE RESEARCH

The following recommendations can be made for future studies:

1) Based on the test results of this research and other reports in the literature, an integrated system with a tactical grade INS could provide acceptable performance in terms of the accuracy of attitude solution, bridging short GPS outages in most low cost navigation and geo-referencing applications like car navigations, UAV control and guidance. The accuracy provided by MEMS INS still has a big gap from that of a tactical grade INS. To fill this gap, further researches both in hardware manufacturing technology (say MEMS) and software algorithms are needed.
2) De-noising of the raw measurements is one possible way to improve integration performance when using low cost INS sensors. The research in Chapter 3 has investigated the plausibility of using a simplified vehicle dynamic model in de-noising process where the dynamics is modelled as an uncorrelated Gauss-Markov process on each axis. To improve the performance, a proper three-dimensional vehicle dynamic model could be employed to investigate the filtering performance. It is anticipated with a more accurate dynamic model, the filtering results would reflect more closely the true dynamics of the vehicle. Adaptive on-line de-noising algorithms could be an interesting topic for further research.

3) Popular adaptive filtering methods used for GPS/INS integration can be roughly classified into three categories, namely covariance scaling, multi-model adaptive estimation, and adaptive stochastic modelling. Each category has its own advantages and disadvantages. As pointed out in Chapter 4, theoretically covariance based on-line stochastic modelling method could estimate directly the covariance matrices of process noise and measurement errors; however in reality, estimation of many unknown variables at the same time with limited external information could easily cause the filtering process to become unstable due to weak observability. On the other hand, covariance scaling method like the one proposed in Chapter 4 is more reliable, robust, and suitable for practical implementations. But using this method, the stochastic parameters can not be tuned individually to reach optimal performance. It would be worthwhile to investigate an adaptive filtering method to draw on the advantages of both categories discussed above. A partial estimation of the covariance matrices instead of the whole matrix elements using a modified on-line stochastic modelling method could possibly be one option.

4) GPS/INS integrated systems can not completely fulfil the requirements raised by ubiquitous positioning and geo-referencing applications in the near future. Hybrid geo-referencing with integration of more sensors is the only solution in response to those challenges in providing geo-reference any where at any time. Additional geo-sensors may include magnetometer, odometer, barometer, WiFi, cell phone, pseudolite (including LOCATA), vision, map matching. Vision or video imaging is a fundamental tool for providing short range positioning and geo-referencing
functions that has been studied intensively in the field of robotics. Simultaneous localization and mapping (SLAM) combined with automatic image feature identification would be an interesting research topic.


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press, Moscow.


PUBLICATIONS DURING THE PHD STUDIES

Referred journal papers:


Referred conference papers:


Abstract referred conference papers:


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Qualifications

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2002-2003: Master of Engineering Science  
            School of Electrical engineering and telecommunications  
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1987-1992: Bachelor of Engineering Science  
            Department of Automation  
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Industry experience

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Awards and Prizes (2004-present)

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- Member of Institute of Electrical and Electronic Engineers (IEEE);
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